Phys 230
Fall 2013

Intermediate Dynamics: Class Exam II

16 October 2013

Name: Solution **Total:** /50

Instructions

- There are 4 questions on 6 pages.
- Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

Speed of light: $c = 3.0 \times 10^8\,\text{m/s}$

Electron Volt: $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

Question 1

An electron in a particle collider travels in a straight line with speed $0.950c$ with respect to a lab frame of reference. The distance traveled by the electron in the lab frame is 1.50×10^3 m. Determine the time taken for this trip as observed in the electron rest frame.

Lab = unprimed

$\Delta x = 1.50 \times 10^3\,\text{m}$ $v = 0.950c$

By definition $v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v} = \frac{1.50 \times 10^3\,\text{m}}{0.950 \times 3.0 \times 10^8\,\text{m/s}} = 5.3 \times 10^{-6}\,\text{s}.$

Electron rest frame = primed. In this frame the two events (begins trip, ends trip) are at the same location. So, the time elapsed is

$$\Delta t_{\text{same}} = \Delta t_{\text{other}}\sqrt{1-u^2/c^2}$$

$$= 5.3 \times 10^{-6}\,\text{s}\sqrt{1-(0.950)^2}$$

$$= 1.6 \times 10^{-6}\,\text{s}$$

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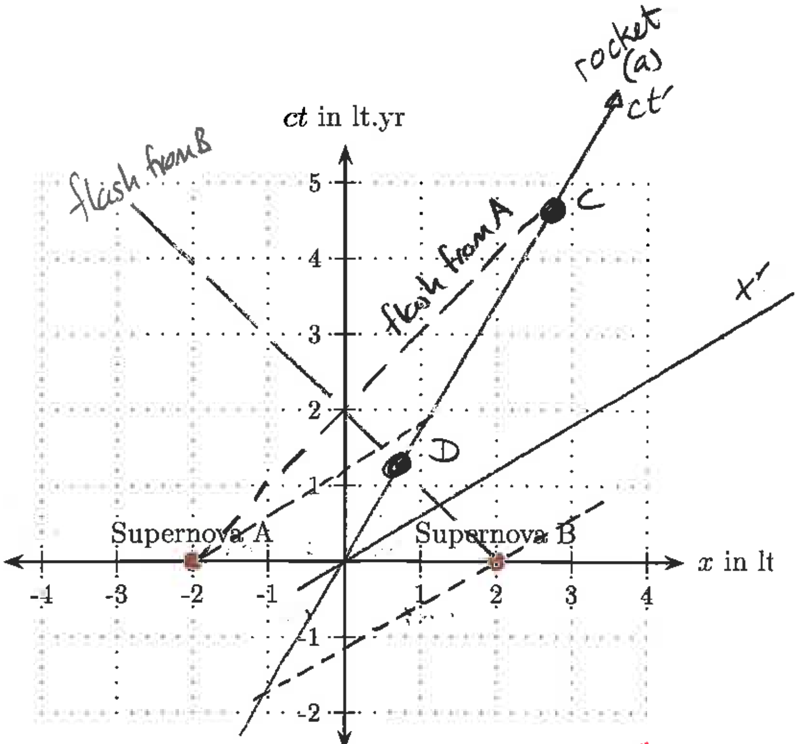
Question 2

An observer in space station is located between two stars, labeled A and B. Each star explodes in a supernova, producing a flash of light, and these events are illustrated on the spacetime diagram in which the observer in the space station uses unprimed coordinates. Another observer in a rocket moves with constant velocity $3c/5$ to the right relative to the space station. The two observers are at the same location when each of their clocks reads 0 yr. This observer uses primed coordinates.



$u = \frac{\Delta x}{\Delta t}$

$\frac{3}{5}c = \frac{\Delta x}{\Delta t}$

$c\Delta t = \frac{5}{3}\Delta x$

↑ slope

a) Indicate the trajectory of the rocket as accurately as possible on the diagram above. 2

b) Indicate the trajectories of the light flashes emitted by each supernova and which travel toward the space station on the diagram above. → slope = ±1 2

c) Which of the following (choose one) is true according to the rocket ship observer? 4

i) Supernova A and B occur at the same time.
ii) Supernova A occurs before supernova B.
[✓ iii) Supernova A occurs after supernova B.]

Explain your answer **using the spacetime diagram.**

Note the x' axis ($ct' = 0$). We can get t' by tracing grid lines parallel to this. We illustrate the lines passing through A, B. That through A intersects at a higher value of ct' ⇒ after that for B

Question 2 continued ...

d) Which of the following (choose one) regarding the arrival of light from the supernova flashes *at the space station* is true? 4

[✓ i)] Light from both supernovae arrives simultaneously according to both observers.
ii) Light from both supernovae arrives simultaneously according to the space station observer but not the rocket observer.
iii) Light from both supernovae arrives simultaneously according to the rocket observer but not the space station observer.

Explain your answer **using the spacetime diagram.**

Clearly the ct value is same for both. So t same for both.
By the gridline construction ct' for A is same as for B. So t' same for both

e) Consider the arrival of light pulses from the supernova *at the rocket.* In what order do they arrive at the rocket according to the space station observer? In what order do they arrive according to the rocket observer? Explain your answer. 4 ~~5~~ 4

The two events are indicated: C = light from A arrives
D = " " B arrives

Clearly the diagram shows that ∴ $ct_C > ct_D \Rightarrow t_C > t_D$
similarly $t_C' > t_D'$
Light from A arrives <u>later</u> at rocket according to <u>both</u>

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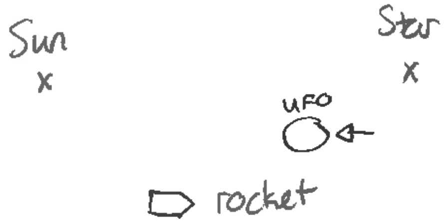

Question 3

The Sun and a star are at rest with respect to each other. A UFO travels at a constant speed from the star to the sun. This is observed by two observers. The first is at rest with respect to the Sun. According to this observer the star is a distance of 40 lt·yr to the right of the Sun. He also observes that the UFO travels with speed $4c/5$ and arrives at the Sun at $t = 50\,\text{yr}$. The second observer travels in a rocket from the Sun to the star with a constant velocity of $3c/5$. At the moment that the rocket observer leaves the Sun both his clock and that of the observer at rest with respect to the Sun read 0 yr.

Sun/Star rest frame = unprimed.

rocket rest frame = primed

a) Determine the time at which the UFO left the star according to the observer at rest with respect to the Sun. 4

Speed of UFO in unprimed

$$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v} = \frac{40\,c\cdot yr}{4c/5} = 50yr$$

$$\Delta t = t_{arrives} - t_{leaves} \Rightarrow t_{leaves} = t_{arrives} - \Delta t = 50yr - 50yr$$

$$\Rightarrow t_{leaves} = 0\,yr$$

b) Determine the times at which the UFO leaves the star *and* at which it arrives at the Sun according to the rocket observer. 6

Event A: UFO leaves

Event B: UFO arrives

Use Lorentz transforms to relate rocket to Sun:

$$t' = \gamma(t - ux/c^2)$$

	t	x
A	0yr	40c·yr
B	50yr	0·cyr

$$t_A' = \gamma(t_A - u/c^2 x_A) = \frac{5}{4}\left(0 - \frac{3}{5\cancel{c}}\, 40\cancel{c}\,yr\right) = -30\,yr$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-(3/5)^2}} = 5/4$$

$$t_B' = \gamma(t_B - u/c^2 \overset{0}{\cancel{x_B}}) = \frac{5}{4}(50yr) = \frac{125}{2}\,yr$$

Question 3 continued . . .

$$v' = \frac{v-u}{1-\frac{uv}{c^2}} = \frac{4/5\,c-(-3/5\,c)}{1+12/25} \quad \boxed{u=-3/5\,c}$$

$$\hookrightarrow = \frac{7}{5}c\,\frac{25}{37} = \frac{35}{37}c$$

c) Which of the following (choose one) is true? ~~3~~ 4

i) The speed of the UFO as observed by the rocket is $c/5$.
ii) The speed of the UFO as observed by the rocket is $7c/5$.
[✓ iii)] The speed of the UFO as observed by the rocket is between $7c/5$ and $c/5$.

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Question 4

A subatomic particle, whose mass is initially at rest. It emits a photon and, after this has mass is 1200 MeV/c^2 and moves with speed $5c/13$.

a) Determine the momentum (in units of MeV/c) and energy (in units of MeV) of the particle after emission of the photon. 6

$$p = \frac{mu}{\sqrt{1-u^2/c^2}} \qquad u = \frac{5c}{13}$$

$$= \frac{1200\,\text{MeV}/c^2 \; 5c/13}{\sqrt{1-(5/13)^2}} = \frac{1200\,\text{MeV}/c \;\; 5/\cancel{13}}{12/\cancel{13}} = 500\,\text{MeV}/c$$

$$\boxed{p = 500\,\text{MeV}/c}$$

$$E^2 = p^2c^2 + m^2c^4 = (500\,\text{MeV})^2 + (1200\,\text{MeV})^2$$

$$= (1300\,\text{MeV})^2 \Rightarrow \boxed{E = 1300\,\text{MeV}}$$

Question 4 continued . . .

b) Determine the energy and momentum of the photon.

4 5

$p_i = p_f = p_{ph} + p_{particle} \Rightarrow p_{ph} = -p_{particle}$

$= -500 \text{ MeV}/c$

(rest $p_i = 0$

$E_{ph} = |p_{ph}| c = 500 \text{MeV}$

c) Which of the following (choose one) is true?

4 3

i) The mass of the particle before photon emission was 1200 MeV/c^2.
ii) The mass of the particle before photon emission was smaller than 1200 MeV/c^2.
[✓ iii)] The mass of the particle before photon emission was larger than 1200 MeV/c^2.

Before, $p=0$ and so

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$E_i^2 = p^2c^2 + m^4c^4 \Rightarrow E_i = mc^2$

Need $E_i = E_f = E_{ph} + E_{particle} = 1300 \text{MeV} + 500 \text{MeV}$

$= 1800 \text{MeV}$

$\Rightarrow m = 1800 \text{MeV}/c^2$