Intermediate Dynamics: Class Exam I

16 September 2013

Name: Solution **Total:** /50

Instructions

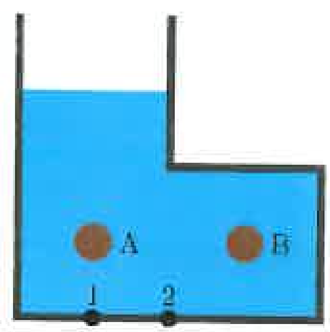
- There are 5 questions on 5 pages.
- Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

$\rho_{\text{water}} = 1.00 \times 10^3\,\text{kg/m}^3$	$1\,\text{atm} = 1.01 \times 10^5\,\text{Pa}$	$T_K = T_C + 273$
$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$	$k_B = 1.38 \times 10^{-23}\,\text{J/K}$	$R = 8.31\,\text{J/mol K}$

Question 1

A container holds water as illustrated. Two identical spherical objects are placed in the water at the illustrated locations.



a) Which of the following is true regarding the pressure at point 1, P_1, and point 2, P_2?

i) $P_1 = P_2$ same depth $\Rightarrow$ same pressure
ii) $P_1 > P_2$
iii) $P_1 < P_2$

b) Which of the following is true regarding the buoyant forces on sphere A, F_A compared to sphere B, F_B ?

i) $F_A = F_B$ displaced volume same buoyant $= \rho_{water} V_{displaced}\, g$
$\Rightarrow F_A = F_B$
ii) $F_A > F_B$
iii) $F_A < F_B$

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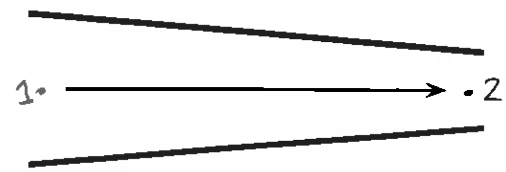
Question 2

$*$ Water flows from left to right through a pipe, with circular cross section as illustrated. The radius of the left is 4.0 cm and on the right 2.0 cm. The pressure at the left is 2.0×10^5 Pa and the speed with which the water enters at the left is 25 m/s. Determine the pressure of the water as it leaves the right end of the pipe (at the level of the middle of the pipe).

2.5 m/s



continuity $V_1 A_1 = V_2 A_2$ +1

$V_1 \pi r_1^2 = V_2 \pi r_2^2$

$V_2 = V_1 \left(\frac{r_1}{r_2}\right)^2 = 4V_1$ +2

+1 Bernoulli $\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$

Same heights $y_1 = y_2 \Rightarrow \frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$

+4

$\Rightarrow P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = P_1 + \frac{1}{2}\rho(v_1^2 - 16v_1^2) = P_1 - \frac{15}{2}\rho v_1^2$

$P_2 = 2.0\times10^5\,\text{Pa} - \frac{15}{2}\,1000\,\text{kg/m}^3\,(2.5\,\text{m/s})^2$

$= 2.0\times10^5\,\text{Pa} - 0.47\times10^5\,\text{Pa}$

$= 1.5\times10^5\,\text{Pa}$

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Question 3

A monoatomic ideal gas undergoes the illustrated process. Which of the following (choose one) is true regarding the thermal energy, ΔE_{th}, and heat flow, Q, for the entire process?

a) $\Delta E_{\text{th}} > 0$ and $Q > 0$.

b) $\Delta E_{\text{th}} > 0$ and $Q < 0$.

c) $\Delta E_{\text{th}} < 0$ and $Q > 0$.

d) $\Delta E_{\text{th}} < 0$ and $Q < 0$.

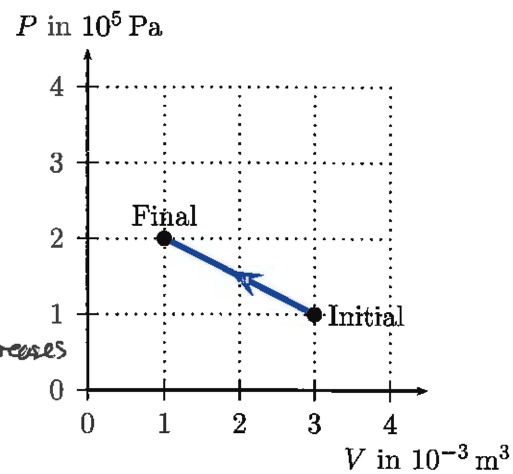
e) $\Delta E_{\text{th}} = 0$ and $Q < 0$.

$\Delta E_{th} = \frac{3}{2} nR\Delta T$

$T = \frac{PV}{nR} \Rightarrow T$ decreases

$\Delta E_{th} < 0$

$\Delta E_{th} = Q + W$



$Q = \Delta E_{th} - W$ but $W > 0$ compression

$\Rightarrow$ neg − pos = neg

$Q < 0$

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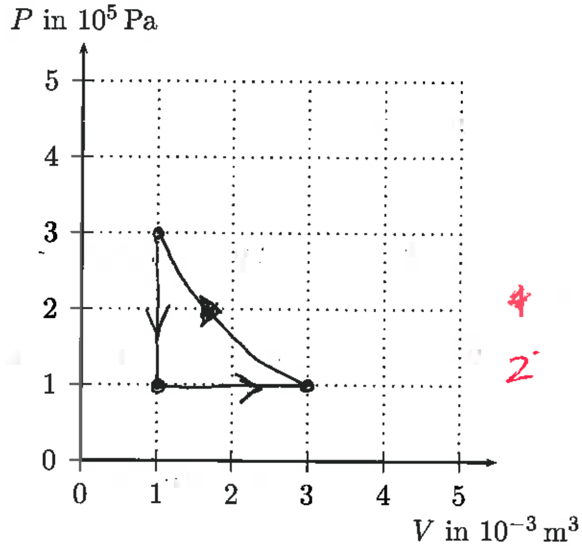
Question 4

A diatomic ideal gas (0.10 mol) is initially at pressure 1.0×10^5 Pa and volume 1.0×10^{-3} m^3. The gas first undergoes an isobaric expansion that triples its initial temperature. It then undergoes an isothermal compression back to its original volume. Finally it undergoes a constant volume process back to its initial state.

a) Sketch the process as accurately as possible on the PV diagram supplied. Determine the volume at the end of the isobaric expansion and pressure at the end of the isothermal compression.



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$PV = nRT \quad V = \left(\frac{nR}{P}\right) T$

isobaric $\nearrow$ constant, T tripled

$\Rightarrow$ V tripled

$\boxed{V = 3 \times 10^{-3} m^3 \text{ at end of isobaric}}$ 1

For isothermal $PV = $ constant. Along this curve $PV = 3 \times 10^2$ J

So at end $V = 1 \times 10^{-3} m^3 \Rightarrow P = \frac{3 \times 10^2 J}{1 \times 10^{-3} m^3} = 3 \times 10^5$ Pa 1

b) Determine the work done on the gas, the change in thermal energy and the heat supplied for *each part* of the process. Enter your results in the table on the next page.

1 $\Delta E_{th} = Q + W$ and, $W = -\int P dV$. For a diatomic gas $E_{th} = \frac{5}{2} nRT$.

But $PV = nRT \Rightarrow$ $E_{th} = \frac{5}{2} PV$ 1

Isobaric P const $\Rightarrow$

$W = -\int P dV = -P \Delta V$

$= -1 \times 10^5 Pa \, (3 \times 10^{-3} - 1 \times 10^{-3}) m^3$

$= -200$ J 1

Question 4 continued . . .

Also $\Delta E_{th} = \frac{5}{2}\Delta(PV) = \frac{5}{2} P\Delta V = \frac{5}{2} 1\times10^5 Pa \times 2.0\times10^{-3} m^3$

$= 500 J$ 2 |

$Q = \Delta E_{th} - W = 500J - (-200J) = 700J$ |

Isothermal: $W = -\int P dV = -\int \frac{nRT}{V} dV = -nRT\int \frac{dV}{V} = -nRT \ln V \Big|_{V_i}^{V_f}$

$= -nRT \ln \frac{V_f}{V_i} = -P_i V_i \ln \frac{V_f}{V_i} = -300J \ln(\frac{1}{3}) = 330J$ 3

$\Delta E_{th} = \frac{5}{2} nR\Delta T = 0$ since $\Delta T = 0$ |

$Q = \Delta E_{th} - W \Rightarrow Q = -W = -330J$ |

Const vol: $W = -\int P dV = 0$ since no volume change. |

$\Delta E_{th} = \frac{5}{2}\Delta PV = \frac{5}{2} V\Delta P = \frac{5}{2} 1\times10^{-3}\times(-2\times10^5) = -500J$ 2 |

$Q = \Delta E_{th} - W = -500J - 0J = -500J$ |

Stage	ΔE_{th}	Q	W
Isobaric expansion	500J	700J.	-200J
Isothermal compression	0J	-330J	330J.
Constant volume process	-500J	-500J	0J
Total	0J	-130J	130J

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Question 5

Two quantities of argon (0.30 mol) and neon (0.20 mol), both monoatomic gases, are mixed. The initial temperatures are 200 K for the argon and 300 K for the neon. The gases are allowed to reach thermal equilibrium after mixing.

a) Which of the following is true (choose one) regarding the average energy per molecule after equilibrium?

$E_{avg} = K_{avg} = \frac{3}{2} k_B T$

T same

i) The average energies are the same.

ii) The average energy for the argon is larger than that for the neon.

iii) The average energy for the argon is smaller than that for the neon.

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b) Determine a relationship between the total energy for the argon and that for the neon after equilibrium.

Monoatomic: $E = \frac{3}{2} nRT$ +1 Same T! +2

$E_{Ar} = \frac{3}{2} n_{Ar} RT$

$E_{Ne} = \frac{3}{2} n_{Ne} RT$

$\frac{E_{Ar}}{E_{Ne}} = \frac{n_{Ar}}{n_{Ne}} = \frac{0.3\,mol}{0.2\,mol}$ +1

$\Rightarrow E_{Ar} = \frac{3}{2} E_{Ne}$ +1

after

c) Determine the total thermal energy of the mixture before mixing.

Before

$E_{Ar} = \frac{3}{2}\, 0.30\,mol \times 8.314\, J/mol{\cdot}K \times 200K = 750\,J$ +1

$E_{He} = \frac{3}{2}\, 0.20\,mol \times 8.314\, J/mol{\cdot}K \times 300K = 750\,J$ +1

$E_{total} = E_{Ar} + E_{He} = 1500\,J$ +1

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$1500 = E_{Ar} + E_{Ne}$

$= \frac{5}{2} E_{Ne}$

$E_{Ne} = 600\,J$

$E_{Ar} = 900\,J$