Intermediate Dynamics: Class Exam II

22 October 2012

Name: _____ **Total:** **/50**

Instructions

- There are 5 questions on 5 pages.
- Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

$$U_{\text{spring}} = \frac{1}{2}\, k(\Delta s)^2$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

Question 1

The motion of an oscillator is represented by the complex function

$$z(t) = (3+3i)\ e^{i4\pi t}$$

a) Determine an expression for the position of the oscillator $x(t)$.

b) Determine the velocity of the oscillator at $t = 0$.

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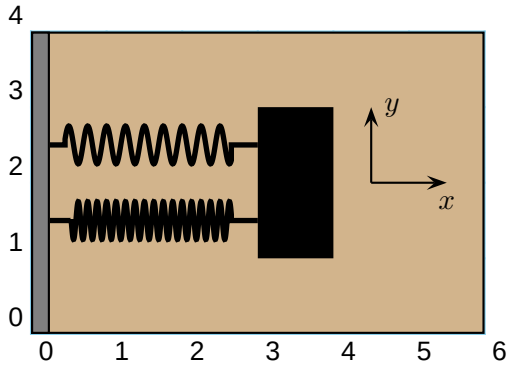
Question 2

A block of mass m is connected to two springs, each of which has the same unstretched length. The block can slide left and right along a horizontal frictionless table. The block is displaced along the illustrated x direction and when it does this the two springs are stretched by the same length. The upper spring has spring constant k_1 and the lower spring k_2. Let x denote the displacement of the block from equilibrium.



a) Let x denote the displacement of the block from equilibrium. Determine an expression for the energy of the system in terms of x and $\frac{dx}{dt}$.

b) Determine an expression for the angular frequency and the period of oscillation of the block.

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Question 3

The charge in an RLC series circuit satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\,q = 0$$

where L is the inductance, R the resistance and C the capacitance.

a) Determine expressions for the undamped (natural) angular frequency, ω_0, the damping constant γ and the quality factor, Q, in terms of R, L and C.

b) Suppose that $L = 3.0 \times 10^{-4}\,\mathrm{H}$, $C = 3.0 \times 10^{-8}\,\mathrm{C}$ and $R = 2.0\,\Omega$. If the total energy in the system is initially E_0, find the amount of time required for the total energy to reach $E_0/4$.

Question 3 continued ...

c) The same system is driven by a time varying potential and the charge satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\,q = V_0\cos(\omega t)$$

where ω is the driving frequency and $V_0 = 5.0\,\mathrm{V}$. For what value of ω would the power absorbed would be a maximum? Determine range of angular frequencies over which the power would be at least half of this maximum value.

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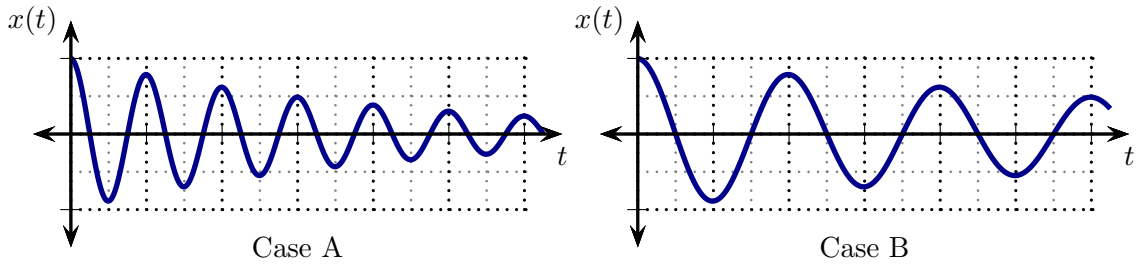
Question 4

Graphs of positions vs. time for two lightly damped oscillators are as illustrated.



a) Which of the following (choose one) is true regarding the damping constants of the oscillators?

i) $\gamma_A = \gamma_B$
ii) $\gamma_A > \gamma_B$
iii) $\gamma_A < \gamma_B$

b) Which of the following (choose one) is true regarding the quality factors of the oscillators?

i) $Q_A = Q_B$
ii) $Q_A > Q_B$
iii) $Q_A < Q_B$

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Question 5

The complex equation of motion for a undamped driven oscillator is

$$\frac{d^2z}{dt^2} + \omega_0^2 z = \frac{F_0}{m} e^{i\omega t}$$

where ω_0 is the natural frequency and ω is the driving frequency. Assume that the solution has the form $z = De^{ut}$ where D and u are complex constants. Find expressions for D, u and the amplitude of oscillation in terms of ω, ω_0, F_0 and m.

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