NMR Quantum Computation with Indirectly Coupled Gates

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Abstract

Existing NMR realizations of two-qubit quantum logic gates have been accomplished "directly" by evolution under scalar coupling between the relevant spins. We use the three carbon spins of labeled alanine to present an NMR realization of a non-trivial two-qubit gate in which information is processed "indirectly" via a chain of scalar couplings between the relevant spins. Such indirect information processing is regarded as essential for the future development of quantum computation. This gate construction enables a successful comprehensive NMR realization of the Deutsch-Jozsa algorithm for functions with three argument bits.

Quantum Logic and Coupled Qubits

Any quantum logic operation can be expressed as a sequence of **fundamental quantum logic operations:**

- Single qubit rotations.

- Two qubit CNOT gates (target spin state flipped conditional on control spin state):

$$a\,|\uparrow\uparrow\rangle + b\,|\uparrow\downarrow\rangle + \mathbf{c}\,|\downarrow\uparrow\rangle + \mathbf{d}\,|\downarrow\downarrow\rangle$$

$$\Updownarrow$$

$$a\,|\uparrow\uparrow\rangle + b\,|\uparrow\downarrow\rangle + \mathbf{d}\,|\downarrow\uparrow\rangle + \mathbf{c}\,|\downarrow\downarrow\rangle$$

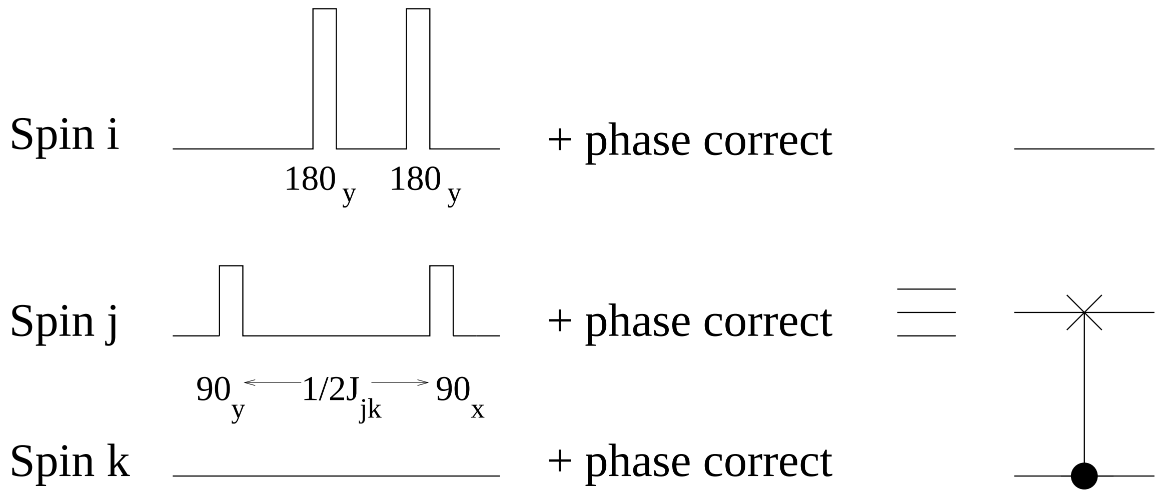
Experimental realizations of CNOT gates are typically much more difficult than those for single bit gates. All proposals for **realizations of CNOT gates require couplings between control and target qubits**.

CNOT realizations

Existing NMR schemes use **J-couplings** between control and target qubits to realize CNOT gates.

▶ "Direct" CNOT:

- Example (control → k, target → j):

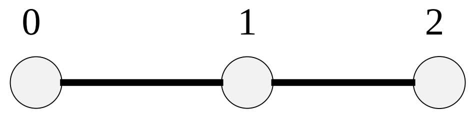


- Realization **directly via evolution under target-control coupling**, J_{jk}.
- All couplings to spin i removed by refocusing.
- **Ineffective if J_{jk} is too small or absent,** i.e. if

$$1/2J_{jk} \gtrsim T_1, T_2.$$

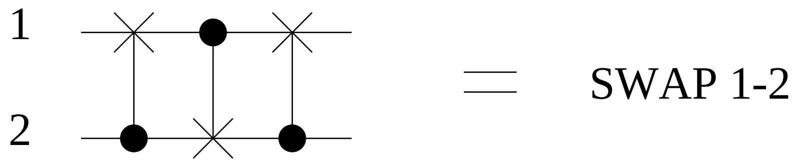
▶ “Indirect” CNOT

- If the control-target J-coupling strength is too weak an indirect realization is possible by swapping spin states via a **chain of sufficiently strong couplings**.
- Example (control → 0, target → 2):

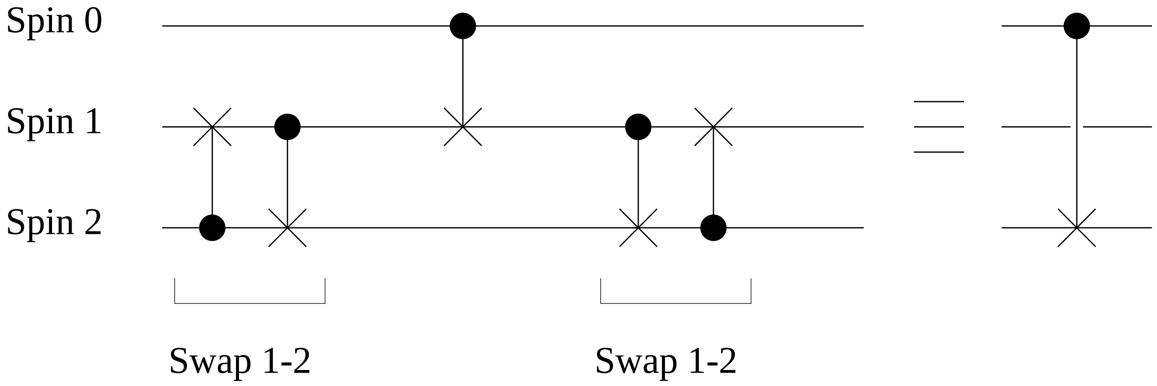


$$\frac{1}{2J_{01}}, \frac{1}{2J_{12}} \ll T_1, T_2; \quad \frac{1}{2J_{02}} \gtrsim T_1, T_2$$

- SWAP gate decomposition:



- Simplified gate array:

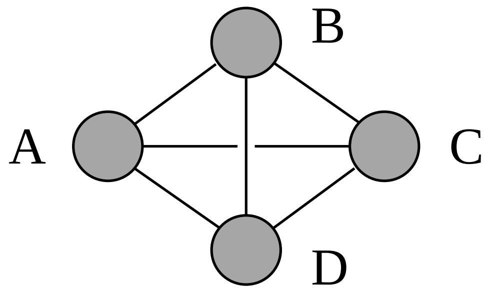


- The indirect CNOT is **independent of J_{02}.**

Coupling Architecture

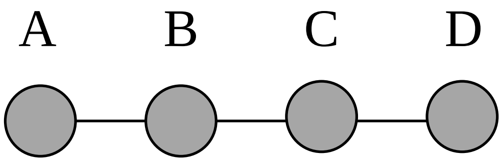
The configuration of useful couplings between qubits dictates the type, efficiency and difficulty of gate realizations.

Full coupling configuration:



- complex refocusing scheme,
- complex spectra,
- all two qubit gates done directly (easy).

Linear coupling configuration:

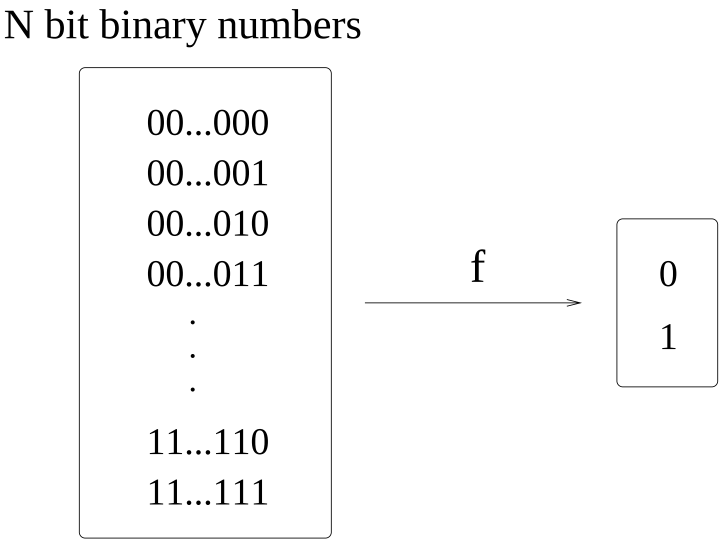


- simple refocusing schemes,
- simple spectra,
- some two qubit gates done indirectly (difficult).

Deutsch Problem

The Deutsch problem and algorithm investigate global properties of certain binary functions.

- Is the function:



  - constant or
  - balanced (returns 0 and 1 the same number of times)?

- Example (balanced function returning rightmost bit):

$$f(x_2x_1x_0) = x_0$$

- **Classical solution:**
  - Evaluate f on an appropriate number of arguments e.g.: $f(0), \ldots, f(2^{N-1}+1)$
  - Number of steps required for a conclusive answer grows as $\exp(N)$.

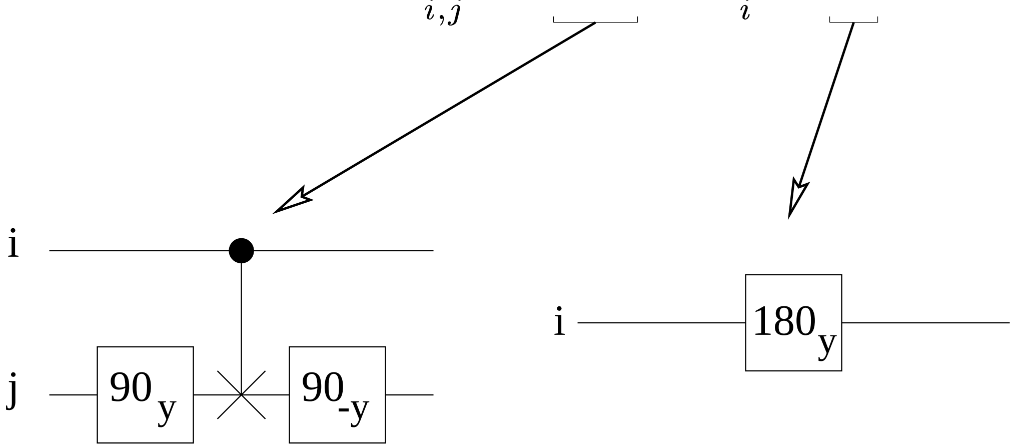
Deutsch-Jozsa Algorithm

There exists an efficient **quantum algorithm** which requires N **qubits** *(Collins et al. (1998))* and which answers the problem conclusively with **one application of the function evaluation operator:**

$$\hat{U}_f \left| x \right\rangle := (-1)^{f(x)} \left| x \right\rangle .$$

- For $N = 3$ expand $\hat{U}_f$ in terms of basic gates via function expansion and concatenation of associated **quadratic and linear term gates:**

$$f(x_2 x_1 x_0) = \sum_{i,j} a_{ij} \underbrace{x_i x_j} + \sum_i a_i \underbrace{x_i} + a$$



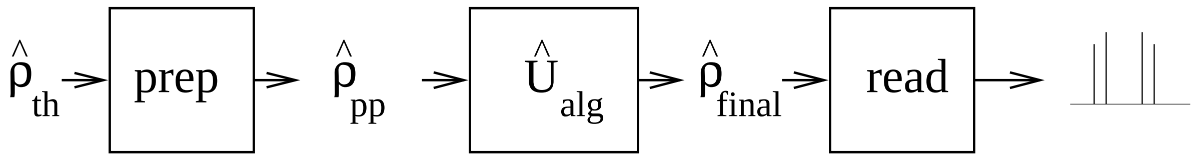
- Example (a balanced function for $N = 3$):

$$f(x_2 x_1 x_0) = x_2 x_1 \oplus x_1 x_0 \oplus x_2 x_0,$$

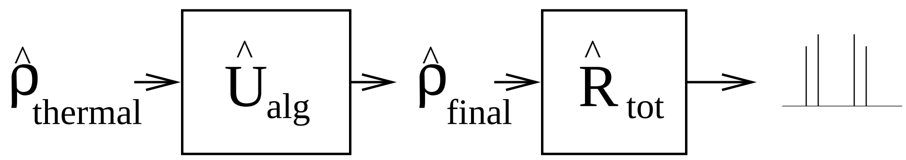
which requires construction of all quadratic term gates.

NMR Deutsch-Jozsa: Theory

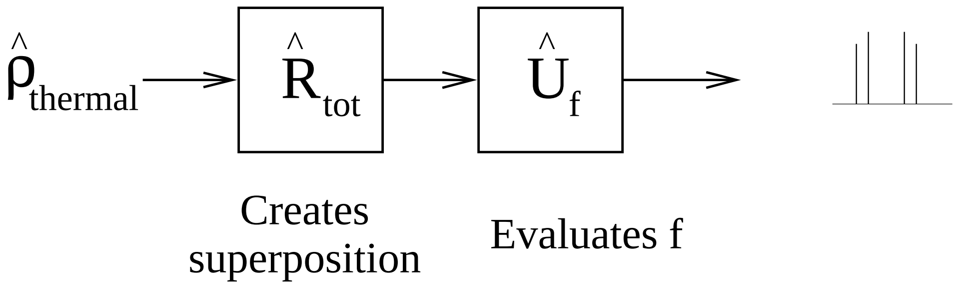
General schemes for NMR quantum computation include a **pseudo-pure state preparation step** and **tomography** via readout pulses.



- **Simplifications** are possible for the Deutsch-Jozsa algorithm in the **range** $N \leq 3$**:**
  - **Omit preparation** steps and use the **simplest readout**:



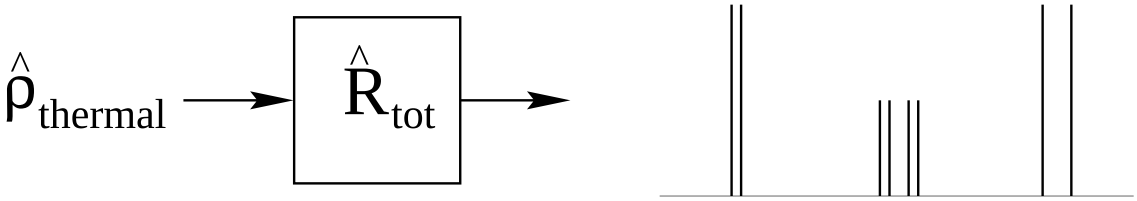
resulting in:



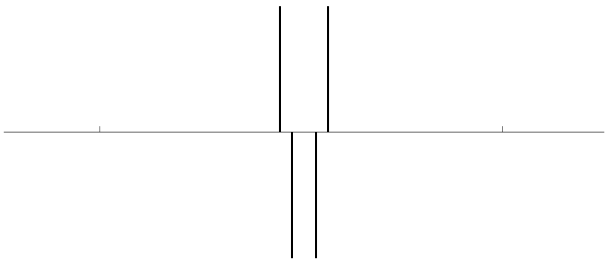
Answer Extraction: $N \leq 3$

The problem may be answered by comparing the spectrum acquired after application of the function evaluation step to a fiducial spectrum.

- **Fiducial** spectrum:



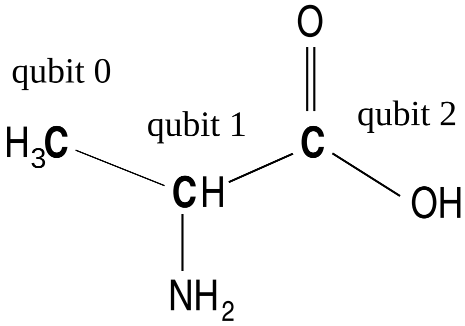
- For each f compare output to fiducial:
  - **all** lines **upright** $\Rightarrow$ **constant**,
  - **any** line **inverted** $\Rightarrow$ **balanced**.
  - example (balanced):



This readout method **requires** that **each spin is coupled to at least one other spin.** This does not impose an additional burden at this level since this is also a necessity for realization of the quantum dynamics in the function evaluation step.

Hardware

For the $N = 3$ DJ algorithm the **three qubits** are provided by the **nuclear spins of ^{13}C labelled alanine.**



- Experiments were conducted using a Bruker DRX-500 spectrometer.
- Hamiltonian for the ^{13}C nuclear spins (proton spins removed by standard decoupling technique):

$$\hat{H} = \frac{1}{2}\sum_{j=0}^{2} \omega_j \hat{\sigma}_z^j + \frac{\pi}{2}\sum_{i \neq j} J_{ij} \hat{\sigma}_z^i \otimes \hat{\sigma}_z^j$$

with coupling strengths:

$$J_{21} = 55\,\mathrm{Hz} \qquad J_{10} = 35\,\mathrm{Hz} \qquad J_{20} = 1.57\,\mathrm{Hz}$$

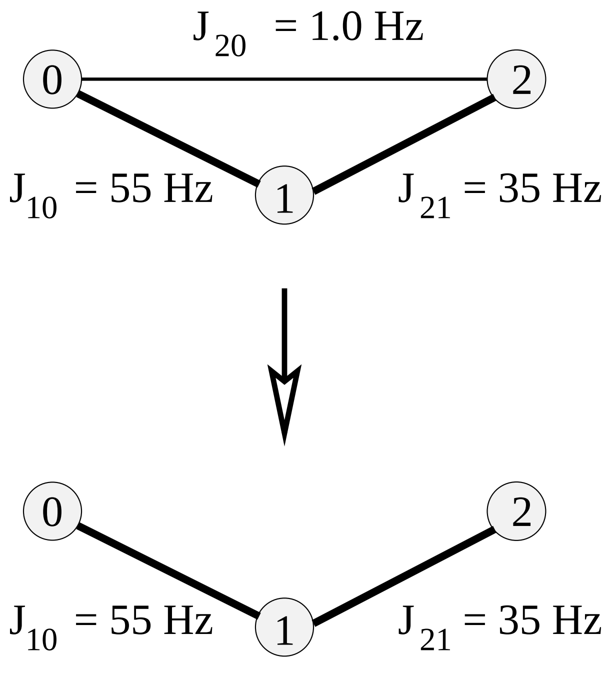
Coupling Architecture

Relaxation times:

- Smallest T_2 or T_1 is $0.41\mathrm{s}$

"Linear" coupling architecture:

- The **qubit 0 - 2 coupling is too weak** to be used for two qubit gates. Elimination via standard refocusing techniques allows for **indirect gates using the linear chain of stronger couplings.**



- Information must be swapped.
- **Faster 0-2 two qubit gates** than with direct realization.

Function Evaluation Gates

Simplified pulse sequence

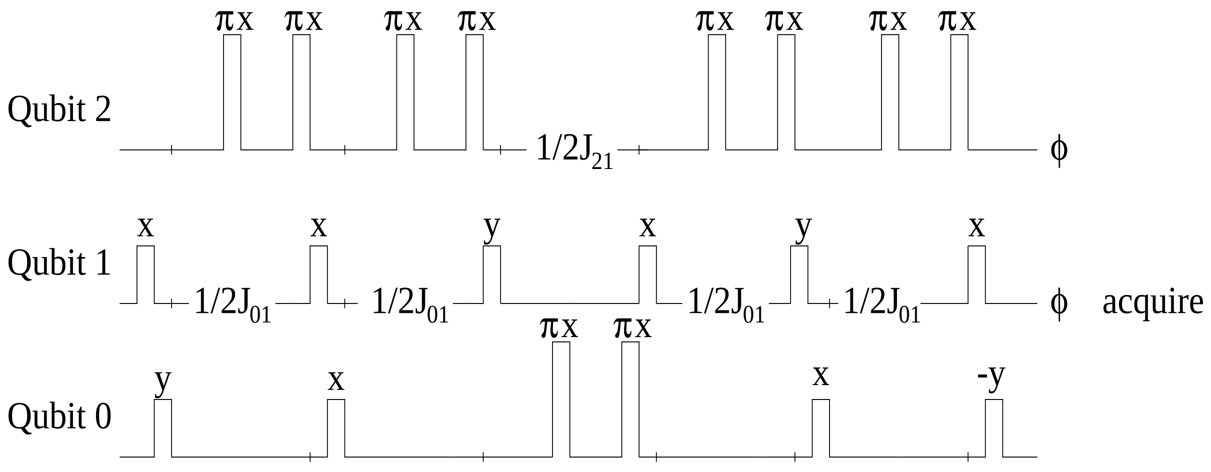
- For the quadratic term ($x_i x_j$):

$$[1/2J_{ij}] - [90]^i_{-z} - [90]^i_{-z}$$

- Produces antiphase doublets (after initial rotation) for qubits i and j.

Indirect quadratic term gate ($x_2 x_0$):

- Use indirect CNOT(2,0) to provide:



All pulses are Gaussian with duration 0.5ms (qubit 2) or 0.7ms (qubits 1 and 0).

- Duration: 70ms (compare to 500ms for direct case).
- Requires no prior information about J_{20}.

Conclusions

Deutsch-Jozsa algorithm development:

- Thermal equilibrium plus simplest readout satisfactory for $N \leq 3$.
- Simple decomposition of $\hat{U}_f$ for $N \leq 3$.

Deutsch-Jozsa experimental realization:

- Successful for all classes of balance functions for $N = 3$.
- Only uses linear chain for information processing.

Indirect two qubit quantum gates

- Indirect CNOT enables faster realization of certain two qubit gates.
- Indirect two qubit gate is independent of coupling strength between the qubits and can provide a means for determining this.
- Possible errors via coupling evolution during selective rotations?