FRW Cosmology & Inflationary Theory

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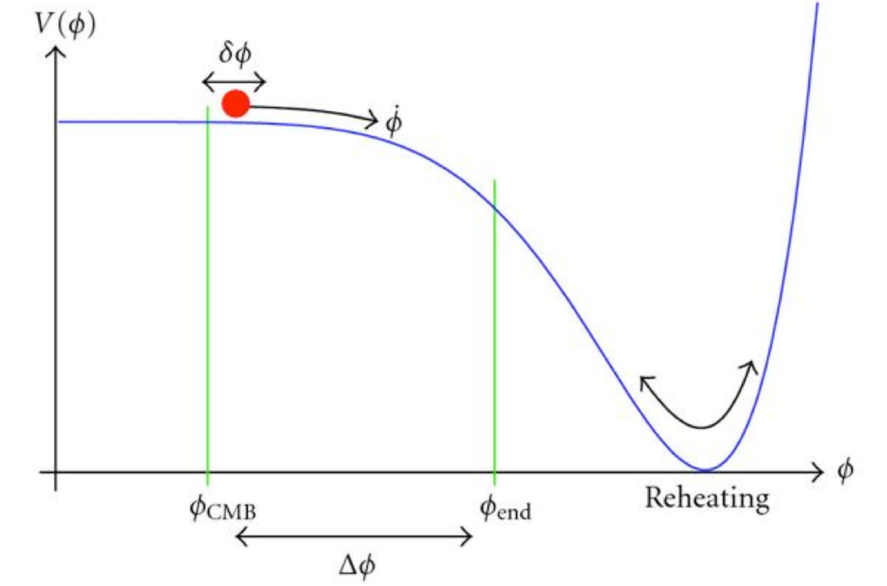
Physics Seminar
Feb. 15, 2018



https://www.hindawi.com/journals/aa/2010/565248/fig1/

Cosmology

- is the scientific study of the large scale properties of the Universe as a whole.

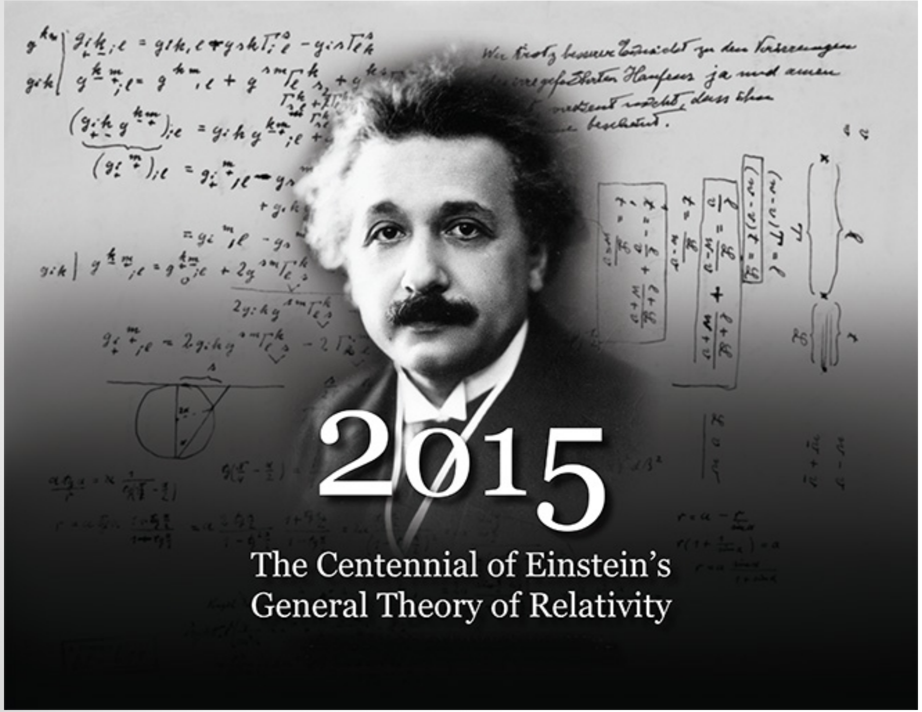
- addresses questions like:
    - Is the Universe (in)finite in spatial extent?
    - Is the Universe (in)finite in temporal extent?
    - What are the possible geometries of the Universe?
    - What is the fate of the Universe?

Conclusion

Theory & observational evidence imply that:

- the Universe is *flat & infinite* in spatial extent
- the Universe began w/ a “Big Bang” 13.8 billion years ago
- a hypothesized *early-time inflationary expansion* explains *most* of the puzzles of FRW cosmology
- the Universe is currently undergoing *accelerated expansion* & is predicted to continue to do so *indefinitely!*

In 1915, Einstein publishes his *General Theory of Relativity*



- https://www.aip.org/sites/default/files/styles/einstein_full/public/einstein_slideshow/einstein-calendar-cover2015.jpg?itok=5q5C8sNe

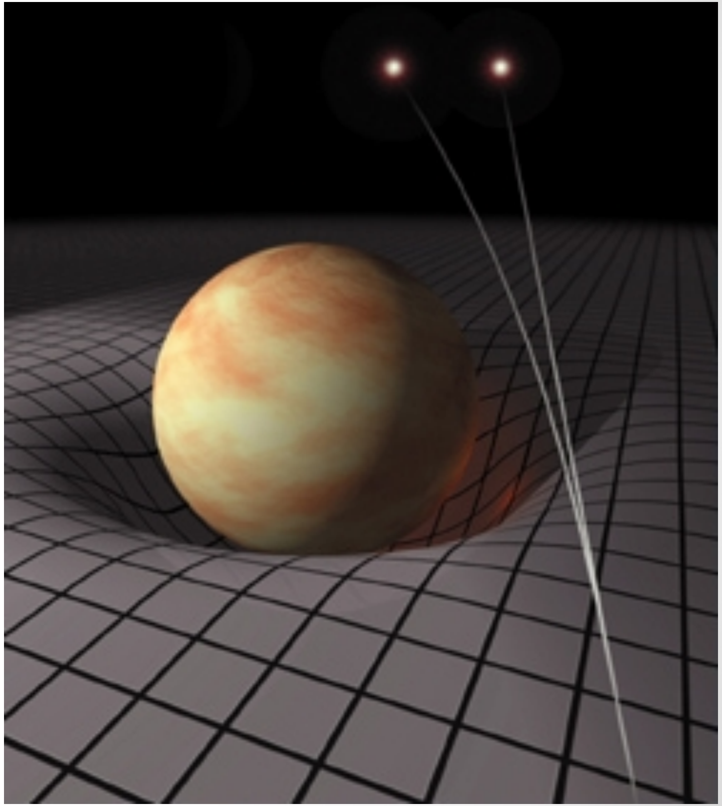
$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

- $G_{\mu\nu}$ describes the *curvature* of spacetime
- $T_{\mu\nu}$ describes the *matter & energy* in spacetime

When forced to summarize the general theory of relativity in one sentence; *time* and *space* and *gravity* have no separate existence from *matter*.

- Albert Einstein

Matter tells *space* how to curve,
Space tells *matter* how to move



Sean M. Carrol, *Spacetime and Geometry: An Introduction to Einstein's General Relativity* (Addison Wesley, 2004)

Friedmann-Robertson-Walker (FRW) cosmology…

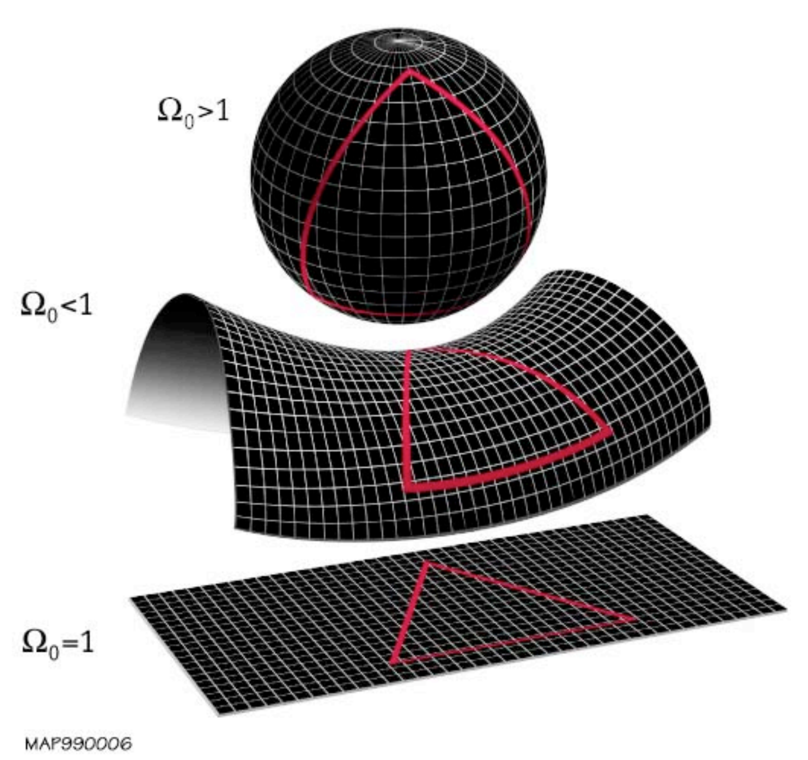
Cosmological Principle

On *sufficiently large distance scales*, the Universe is

1. spatially *isotropic*
2. spatially *homogeneous*

$\Rightarrow$ *Maximally symmetric space*

For a ***Homogeneous*** & ***Isotropic*** Universe…
3 possible geometries



http://wmap.gsfc.nasa.gov/media/990006/990006_2048.jpg

Recent data indicates that the Universe is *flat*

FRW cosmology

$$\rho = 3\frac{\dot{a}^2}{a^2} + 3\frac{\kappa}{a^2}$$

$$p = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \frac{\kappa}{a^2}$$

$$0 = \dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a}$$

- The density (ρ), pressure (p), and curvature (κ) of the Universe determine the *time evolution* of the scale factor (a).
- The matter & energy content consists of
  - *radiation*, *pressureless matter* & *vacuum energy*.

FRW cosmology

Choose an *equation of state* for each fluid component...

$$p = w\rho$$

- For *radiation*: $w = 1/3$
- For *pressureless matter*: $w = 0$
- For *vacuum energy*: $w = -1$

FRW cosmology

Using our *equation of state*,
the *Conservation of Energy equation* can be solved…

$$\rho(a) = \frac{\rho_0}{a^{3(1+w)}}$$

- For *radiation*: $w = 1/3$
- For *pressureless matter*: $w = 0$
- For *vacuum energy*: $w = -1$

The *total density* as a function of the *scale factor* is…

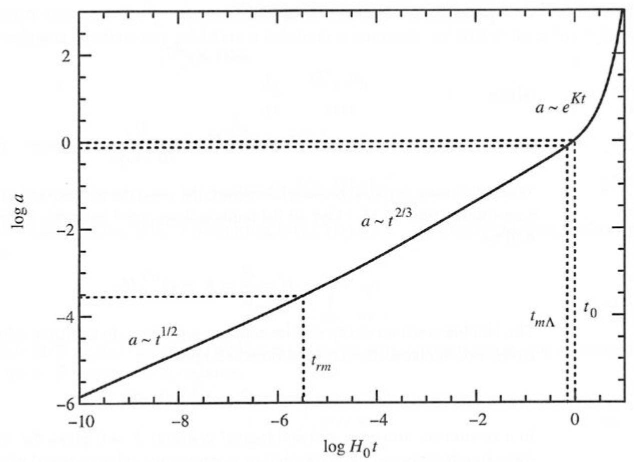
$$\rho(a) = \rho_{\rm crit}\left(\Omega_v + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$$

where $\Omega_i = \dfrac{\rho_i}{\rho_{\rm crit}}$

Here we've set $\kappa = 0$, inline with observation

FRW cosmology

Using our *equation of state*,
the *FRW eqns* can be solved for 1 fluid…



$$a(t) = a_0 t^{2/3(1+w)} \quad \text{for } w \neq -1$$

$$= e^{H(t-t_0)} \quad \text{for } w = -1$$

- For *radiation dominated:* $a(t) \sim t^{1/2}$
- For *matter dominated:* $a(t) \sim t^{2/3}$
- For *vacuum energy dominated:* $a(t) \sim e^{Ht}$

NOTICE:
Early-time, *decelerated* expansion
As $t \to 0$, $a(t) \to 0$ ∴ Big Bang!

Late-time, *accelerated* expansion

FRW cosmology

The *constant* a_0 can be chosen so that...

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3(1+w)}$$

so that $a(t_0) = 1$ where t_0 is the *present time*.
The *total density*, at the present time, becomes...

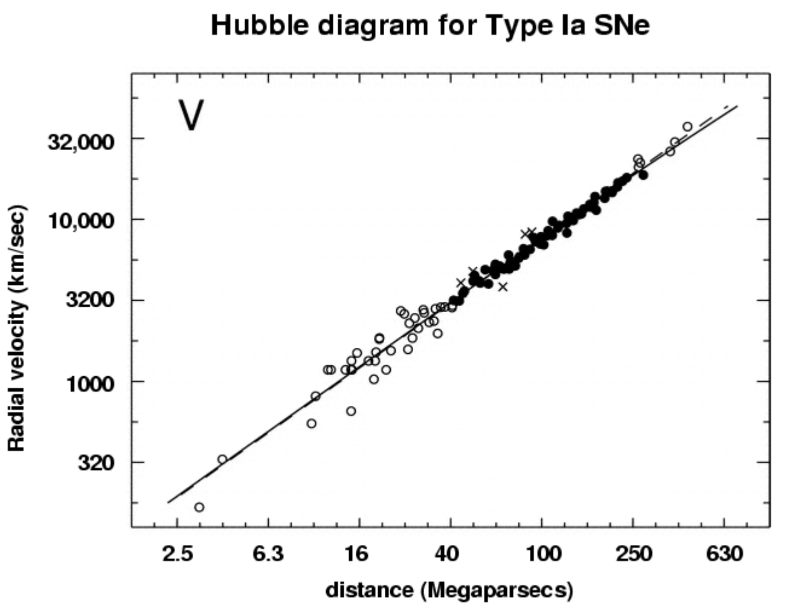
$$\rho(a) = \rho_{\text{crit}} \left(\Omega_{v,0} + \Omega_{m,0} + \Omega_{r,0}\right)$$ where

$$\Omega_{r,0} \sim 10^{-4}$$
$$\Omega_{m,0} = 0.309$$
$$\Omega_{v,0} = 0.691$$

The Hubble Parameter…

Observational evidence implies…

$$H_0 = 72 \pm 7\ (\mathrm{km/s})/\mathrm{Mpc}$$
$$= 2.3 \times 10^{-18}\ \mathrm{s}^{-1}$$



Reindl et al., ApJ 624, 532 (2005)

The *Hubble parameter* can be calculated *theoretically*…

$$H_0 = \frac{\dot{a}(t_0)}{a(t_0)} = \frac{2}{3(1+w)}\frac{1}{t_0} \equiv \frac{1}{t_H}$$

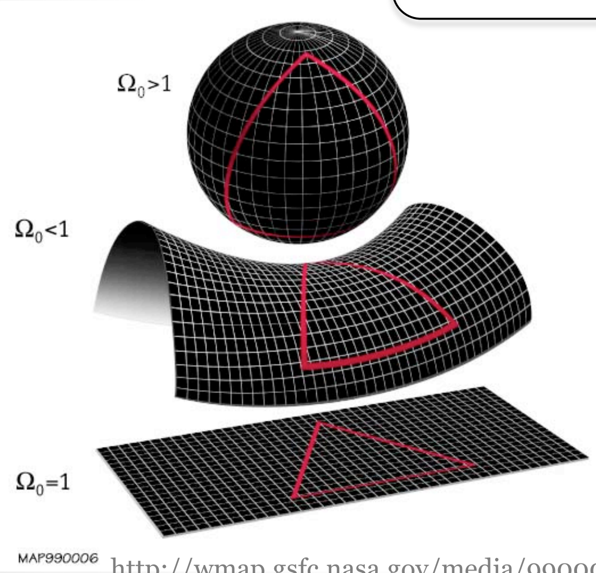
using $a(t) = a_0 t^{2/3(1+w)}$

14×10^9 years

The *Critical Density* of the Universe...

For a *flat* Universe evaluated *now*,
the 1st FRW equation yields the *critical density*...

$$\boxed{\rho_{\rm crit} = \frac{3}{8\pi}\frac{c^2 H^2}{G}} = 8.8 \times 10^{-10}\ \mathrm{J/m^3}$$



http://wmap.gsfc.nasa.gov/media/990006/990006_2048.jpg

$$\rho > \rho_{\rm crit} \quad \therefore \quad \kappa > 0 \quad \therefore \quad \text{closed}$$

$$\rho < \rho_{\rm crit} \quad \therefore \quad \kappa < 0 \quad \therefore \quad \text{open}$$

$$\rho = \rho_{\rm crit} \quad \therefore \quad \kappa = 0 \quad \therefore \quad \text{flat}$$

Puzzle 1:
The Cosmological Constant Problem?

From the *zero-point energies* of vacuum fluctuations…

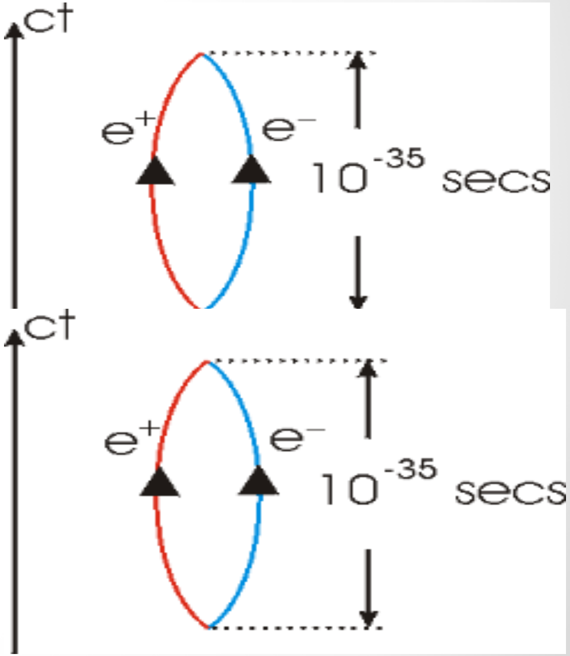
$$\rho_v^{th} \sim 2 \times 10^{109}\ \mathrm{J/m^3}$$

Cosmological observations imply…

$$\rho_v^{obs} \sim 2 \times 10^{-11}\ \mathrm{J/m^3}$$

The ratio yields…

$$\rho_v^{th} / \rho_v^{obs} \sim 10^{120}$$



http://www.upscale.utoronto.ca/GeneralInterest/Harrison/BlackHoleThermo/VirtualPair.gif

Puzzle 2: Flatness Puzzle

Why is the Universe so *flat*?

The 1st FRW equation can be written as

$$\Omega(t) - 1 = \frac{\kappa}{\dot{a}^2} \qquad \therefore$$

where the *density parameter* $\Omega(t)$ ...

$$\Omega(t) \equiv \frac{\rho(t)}{\rho_{\text{crit}}} \quad \text{where } \rho_{\text{crit}} \equiv 3\frac{\dot{a}^2}{a^2}$$

Puzzle 2: Flatness Puzzle

Why is the Universe so *flat*?

The 1st FRW equation can be written as

$$\Omega(t) - 1 = \frac{\kappa}{\dot{a}^2} \quad \therefore \quad \Omega(t) - 1 \propto \kappa a(t)^{1+3w}$$

using $a(t) = a_0 t^{2/3(1+w)}$

where the *density parameter* $\Omega(t)$...

$$\Omega(t) \equiv \frac{\rho(t)}{\rho_{\text{crit}}} \quad \text{where } \rho_{\text{crit}} \equiv 3\frac{\dot{a}^2}{a^2}$$

Puzzle 2: Flatness Puzzle

For a *matter* or *radiation* dominated Universe...

$$|\Omega(t) - 1|_m \propto \kappa a \propto \kappa t^{2/3}$$
$$|\Omega(t) - 1|_r \propto \kappa a^2 \propto \kappa t$$

$\therefore$ LHS must *diverge* w/ time, $\Omega = 1$ is an *unstable fixed point*!

From observation:

1 part in 30 trillion!

$$|\Omega_0 - 1| \leq 0.2 \quad \rightarrow \quad |\Omega_{\rm BBN} - 1| \leq 3 \times 10^{-14}$$

Now

Big Bang Nucleosynthesis

Puzzle 3: Cosmic Coincidence Problem?

Why is the vacuum energy density the *same* order of magnitude as the matter density?

Mathematically:

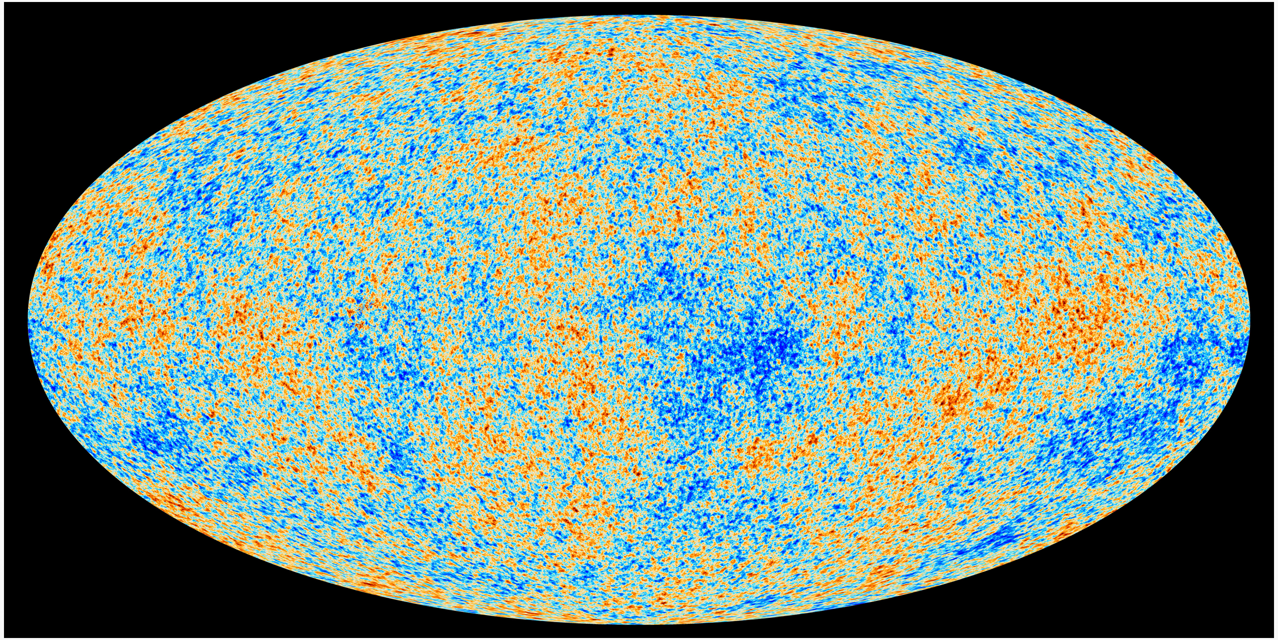
Why is $\Omega_{v,0}/\Omega_{m,0} \sim 1$?

In general:

$$\frac{\Omega_v}{\Omega_m} \propto a^3(t)$$

Puzzle 4: Horizon Problem?

Why is the CMB uniform on large scales ?



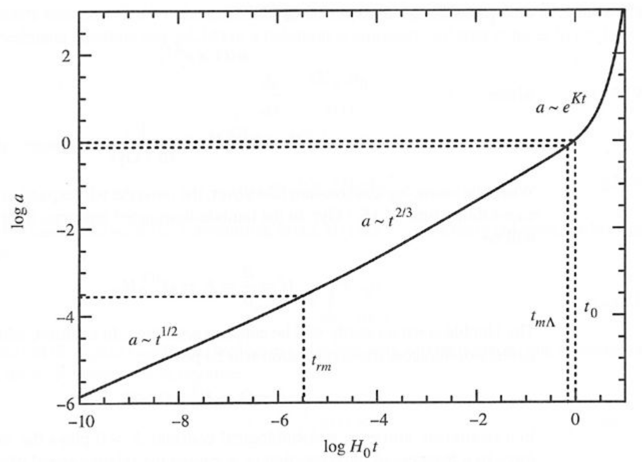
2 antipodal CMB photons are *causally disconnected* yet have

- *same temperature* to 1 part in 10^5! Why?

Puzzle 5:
Initial Jump Start?

The 2^{nd} FRW equation can be written

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(1+3w)\rho$$

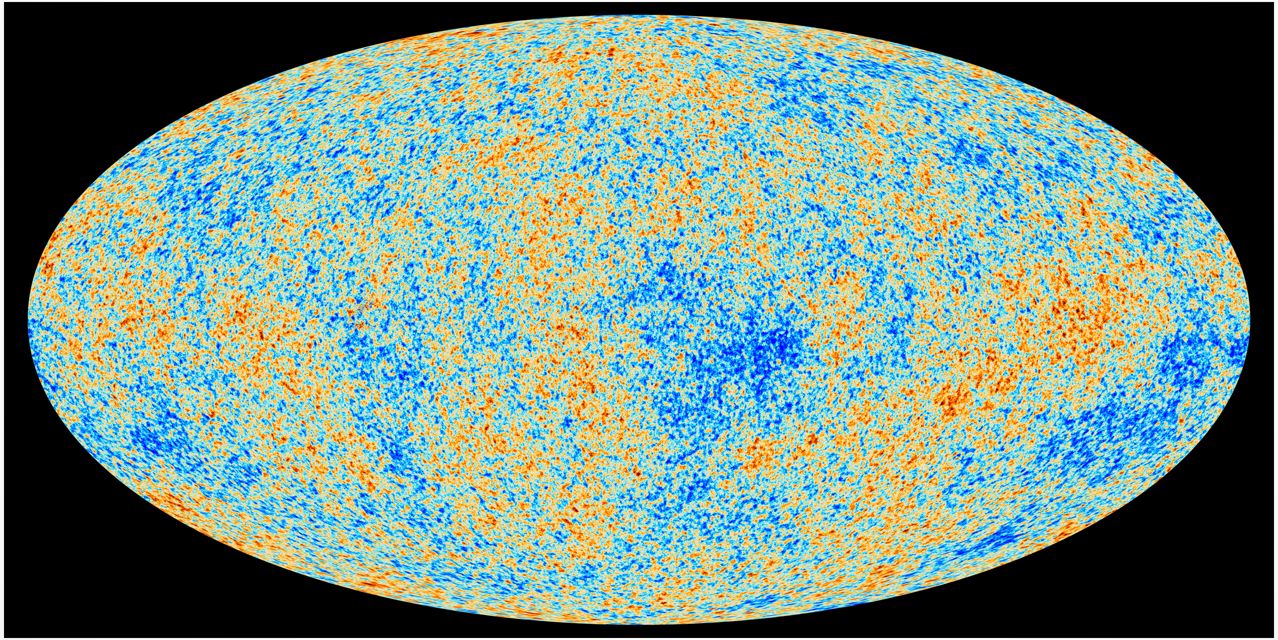


For a *radiation dominated Universe:* $w = 1/3$

So what got FRW cosmology going in the 1^{st} place?

Puzzle 6: Structure of the Universe?

Where did the regions of slightly enhanced density in the early Universe originate from?

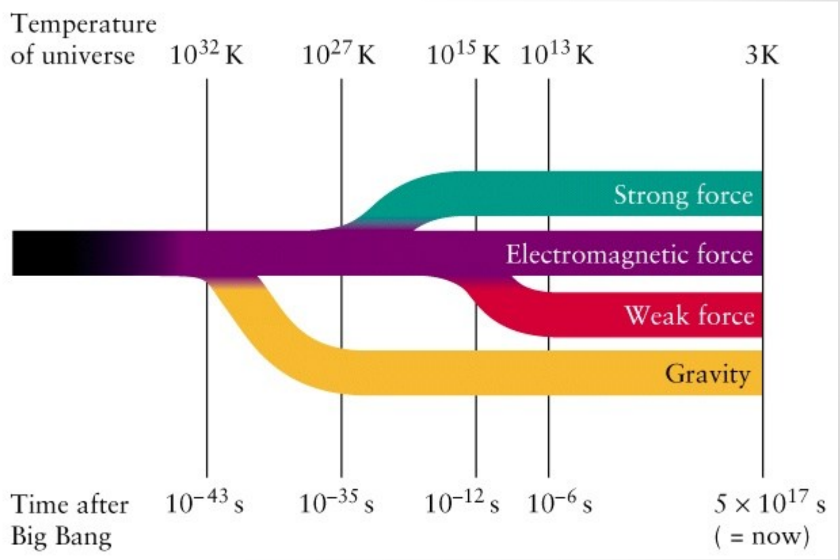


The Inflationary Solution

In 1981, Alan Guth realizes that *most* of these *cosmological* puzzles could be solved by considering *particle* physics.

Grand Unified Theory (GUT) = unification of 3 of 4 fundamental forces

In early Universe, strong force *"freezes"* out releasing enormous amounts of energy

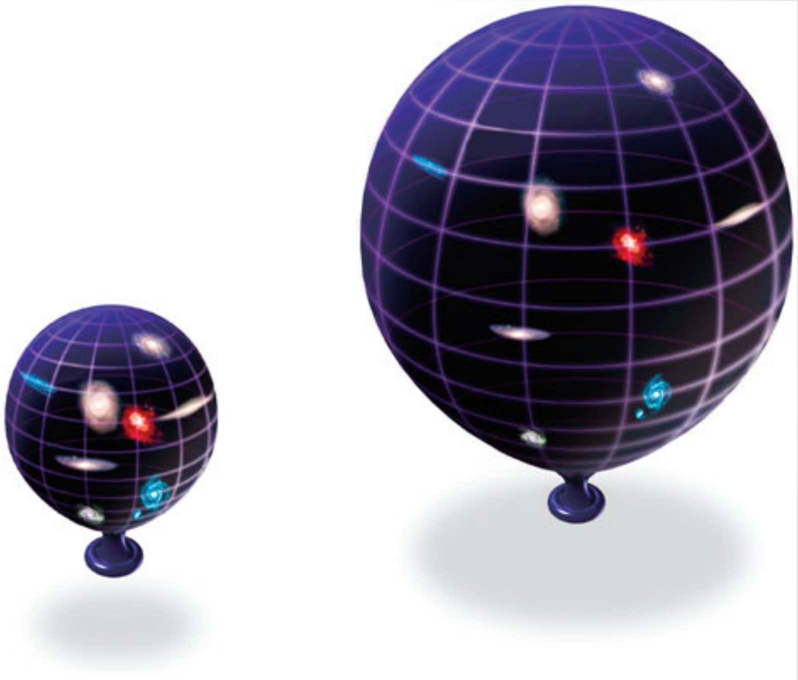


The Inflationary Solution

Supplement to Big Bang Cosmology

Most Big Bang puzzles are solved by this *one* beautifully simple idea.

Early universe is hypothesized to expand by a factor of ~10^{43} in about 10^{-34} s!



Inflationary Theory

For the Robertson-Walker metric and a homogeneous scalar field…

The scale factor, *a(t)*, & the scalar field, *ϕ(t)*, are determined by

$$V(\phi) = 3\frac{\dot{a}^2}{a^2} - \frac{1}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{dV}{d\phi}$$

Inflationary Theory

For the Robertson-Walker metric and a homogeneous scalar field…

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$$V(\phi) = 3\frac{\dot{a}^2}{a^2} - \frac{1}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{dV}{d\phi}$$

Notice:

- Mimics Newton's 2nd Law for a particle being accelerated by a force, $-\frac{dV}{d\phi}$, but impeded by a velocity-dependent retarding force!

Inflationary Theory

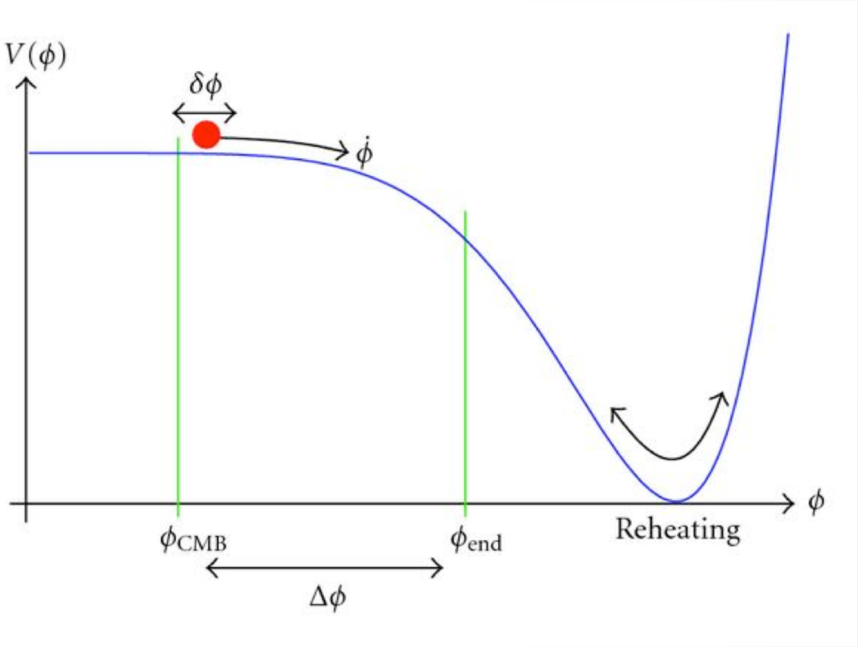
The scalar field reaches "terminal velocity" when...

$$3\frac{\dot{a}}{a}\dot{\phi} = -\frac{dV}{d\phi}$$

For "Slow roll" inflation...

$$V(\phi) \simeq \text{const} \quad \therefore \quad \frac{dV}{d\phi}, \dot{\phi}$$ are small

so $$a(t) \propto e^{H_i t}$$



https://www.hindawi.com/journals/aa/2010/565248/fig1/

Inflationary Theory

During inflation…

- the scale factor is given by…

$$\boxed{a(t) \propto e^{H_i t}} \quad \text{where} \quad t_i < t < t_f$$

Inflationary Theory

During inflation...

- the scale factor is given by...

assume $t_i = t_{GUT} \simeq 10^{-36}$ s

$$\boxed{a(t) \propto e^{H_i t}}$$ where $t_i < t < t_f$

Inflationary Theory

During inflation...

- the scale factor is given by...

assume $t_i = t_{GUT} \simeq 10^{-36}$ s

$$\boxed{a(t) \propto e^{H_i t}}$$ where $t_i < t < t_f$

- the scale factor inflates rapidly...

$$\frac{a(t_f)}{a(t_i)} = e^{H_i(t_f - t_i)}$$ where $H_i \simeq \frac{1}{t_{GUT}} \simeq 10^{36}\ \mathrm{s}^{-1}$

Inflationary Theory

During inflation…

- the scale factor is given by…

assume $t_i = t_{GUT} \simeq 10^{-36}$ s

$$\boxed{a(t) \propto e^{H_i t}} \quad \text{where} \quad t_i < t < t_f$$

- the scale factor inflates rapidly…

$$\frac{a(t_f)}{a(t_i)} = e^{H_i(t_f - t_i)} \quad \text{where} \quad H_i \simeq \frac{1}{t_{GUT}} \simeq 10^{36}\ \text{s}^{-1}$$

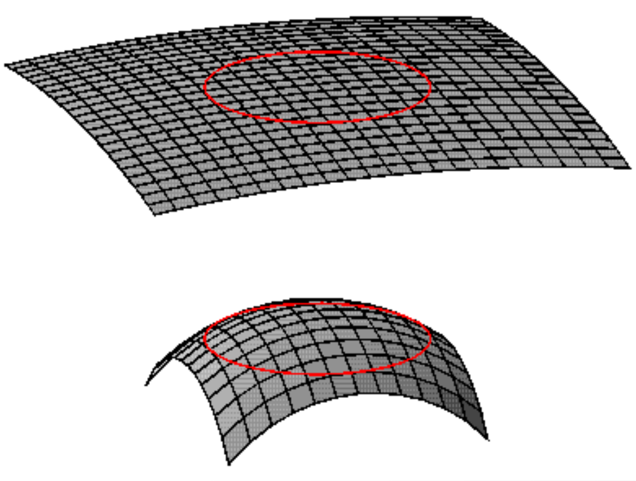
- Now, if $t_f \simeq 10^{-34}$ s then $\dfrac{a(t_f)}{a(t_i)} = e^{100} \sim 10^{43}$!

Puzzle 2: Flatness Puzzle, revisited…

Inflation drives the Universe towards *flatness*

$$\Omega(t) - 1 = \frac{\kappa}{\dot{a}^2} \quad \therefore \quad |\Omega(t_f) - 1|_i \propto \kappa e^{-2H_i t_f} \sim \kappa e^{-200}$$

using $a(t) \propto e^{H_i t}$



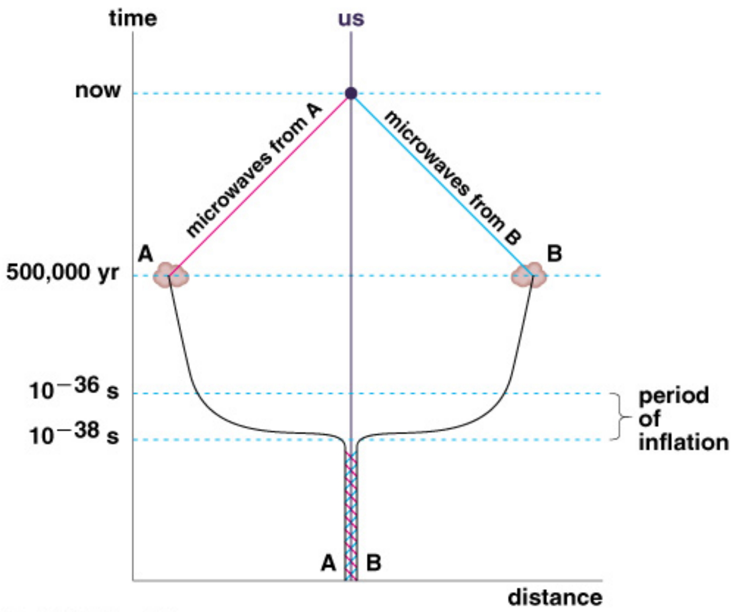
Notice:

$\Omega = 1$ is an attractor for large *t*!

Puzzle 4: Horizon Problem, revisited…

During Inflation, the Universe undergoes *superluminal* expansion to *acausal distances*

The Universe is *smooth* on large scales because it was close together & in a state of *thermal equilibrium* prior to inflation



Puzzle 5: Initial Jump Start, revisited…

The scale factor during Early-time Inflation takes the form…

$$a(t) \propto e^{H_i t}$$

$$\therefore \quad \boxed{\ddot{a}(t) > 0}$$

Inflation got FRW cosmology going in the 1st place!

Puzzle 6:
Structure of the Universe, revisited…

Scalar field Inflationary theory predicts…

- regions of *slightly enhanced* density are due to *quantum fluctuations*

$$\phi(t) \to \phi(t) + \delta\phi(\mathbf{x}, t)$$

- Inflationary theory predicts that these *quantum fluctuations* inflate to yield *gravitational instabilities*!

Conclusion

Theory & observational evidence imply that:

- the Universe is *flat & infinite* in spatial extent
- the Universe began w/ a "Big Bang" 13.8 billion years ago
- a hypothesized *early-time inflationary expansion* explains most of the puzzles of FRW cosmology
- the Universe is currently undergoing *accelerated expansion* & is predicted to continue to do so *indefinitely!*

FRW cosmology

Assumption 1:

Choose the Robertson-Walker metric*…

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Assumption 2:

3 non-interacting, *perfect fluid* components:

- radiation
- *pressureless* matter
- *constant* vacuum energy (a.k.a. *cosmological constant)*

* the Robertson-Walker metric describes a *spatially homogeneous, isotropic* Universe evolving in time

In 1965, observational evidence for the Big Bang!

Arno Penzias & Robert Wilson

- Bell Lab Physicists calibrating the Bell Labs microwave antenna designed for satellite communications
  - Awarded the 1978 Nobel Prize in physics *for their discovery of the cosmic microwave background radiation.*

http://cosmictimes.gsfc.nasa.gov/online_edition/1965Cosmic/images/penzias_wilsonpgg.

Does this *background radiation* have a *Blackbody Spectrum?*

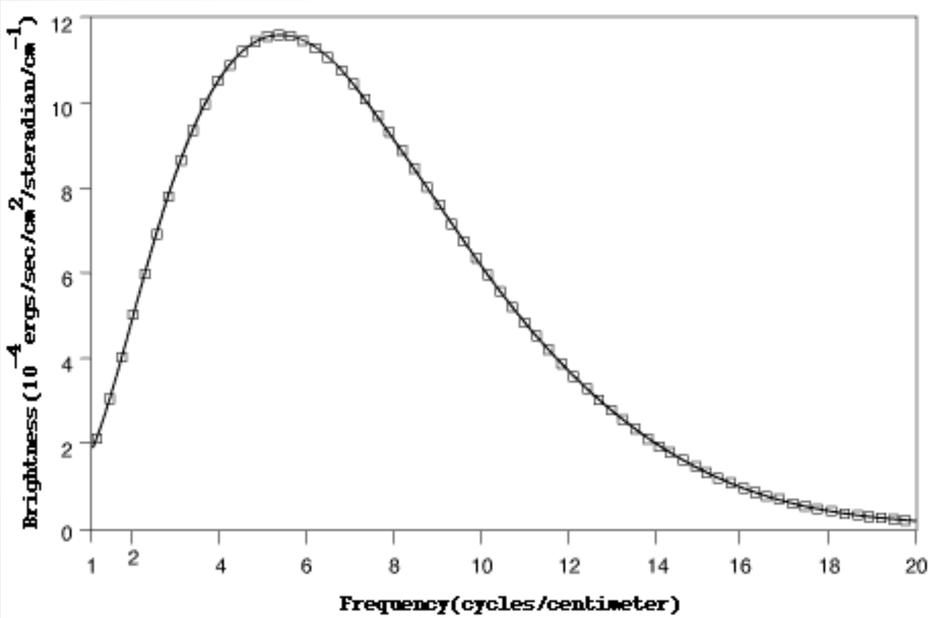
In Nov`89, NASA launches the *Cosmic Background Explorer* (COBE) to measure…

- the spectrum
- the anisotropies

http://cosmos.lbl.gov/Images/cobe_universe.jpg

of the cosmic background radiation.

Spectrum of the *Cosmic Microwave Background Radiation*



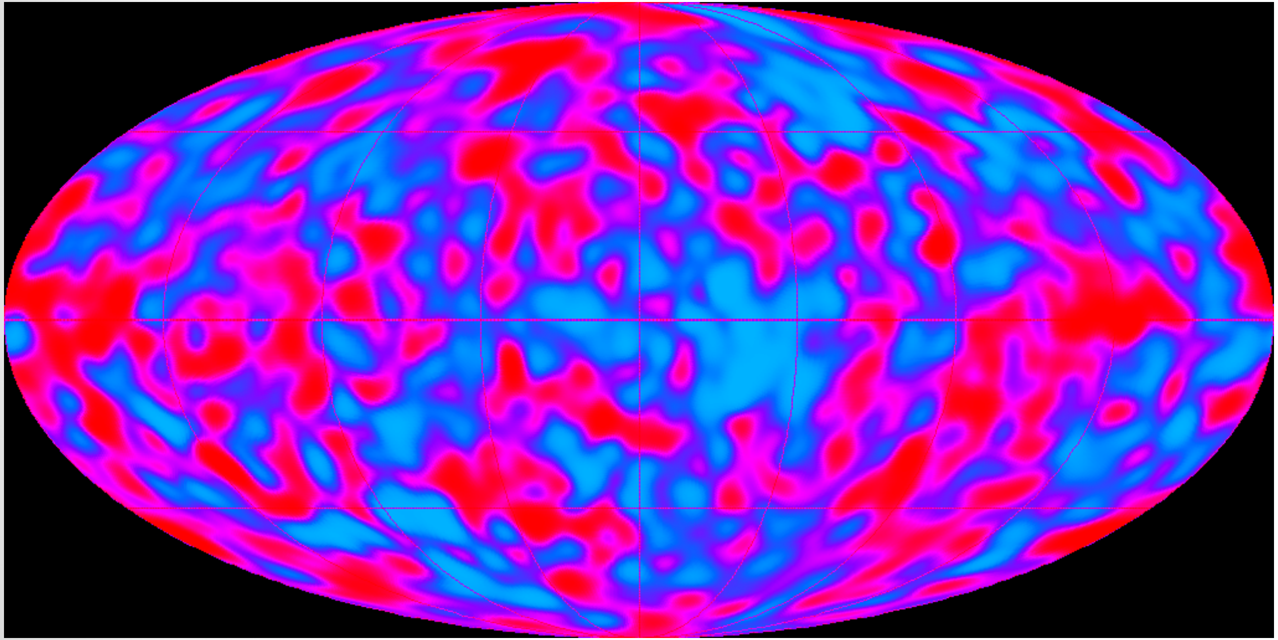
http://www.faculty.umb.edu/gary_zabel/Courses/Parallel%20Universes/Texts/Remote%20Sensing%20Tutorial%20Page%20A-9.htm

John Mather & George Smoot

Awarded the 2006 Nobel Prize in physics for their *discovery of the blackbody form and anisotropy of the cosmic microwave background radiation* measured by COBE.

- The excellent agreement with Planck's law is *the* best fit ever measured!

COBE image of the *Cosmic Microwave Background Radiation*

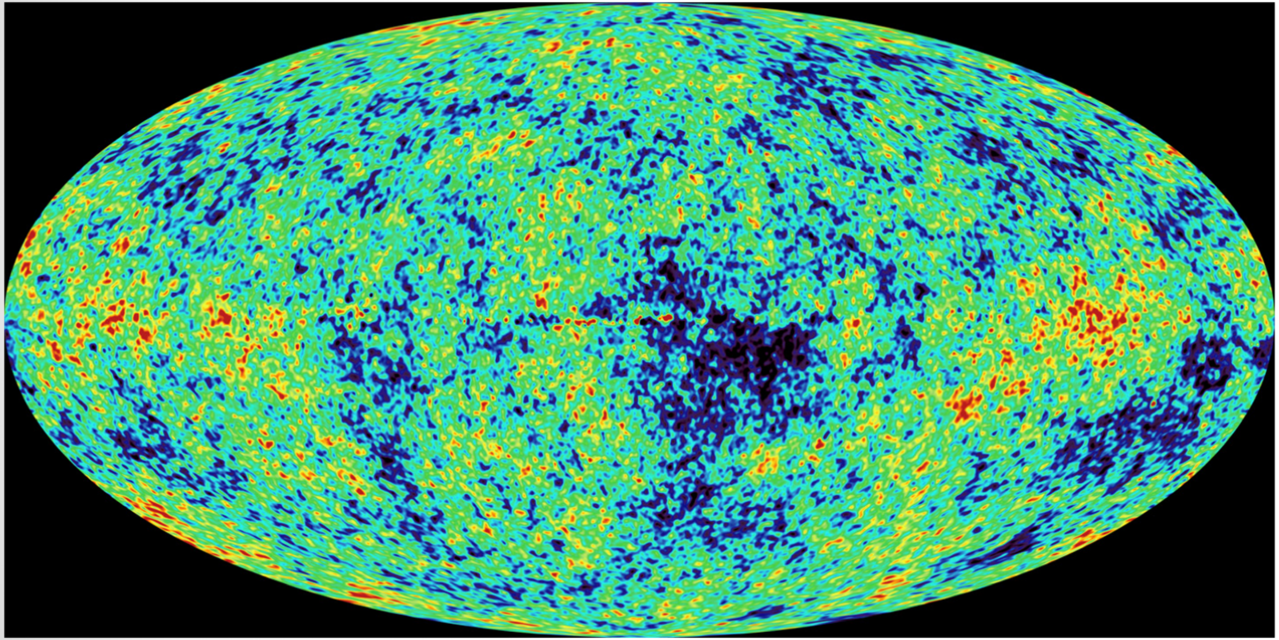


http://www.nasa.gov/images/content/403322main_COBEallsky_full.jpg

- Light from when the Universe was 380,000 years old.
- Map of μK *anisotropies*

$$T_B = 2.725\text{K} \pm 18\mu\text{K}$$

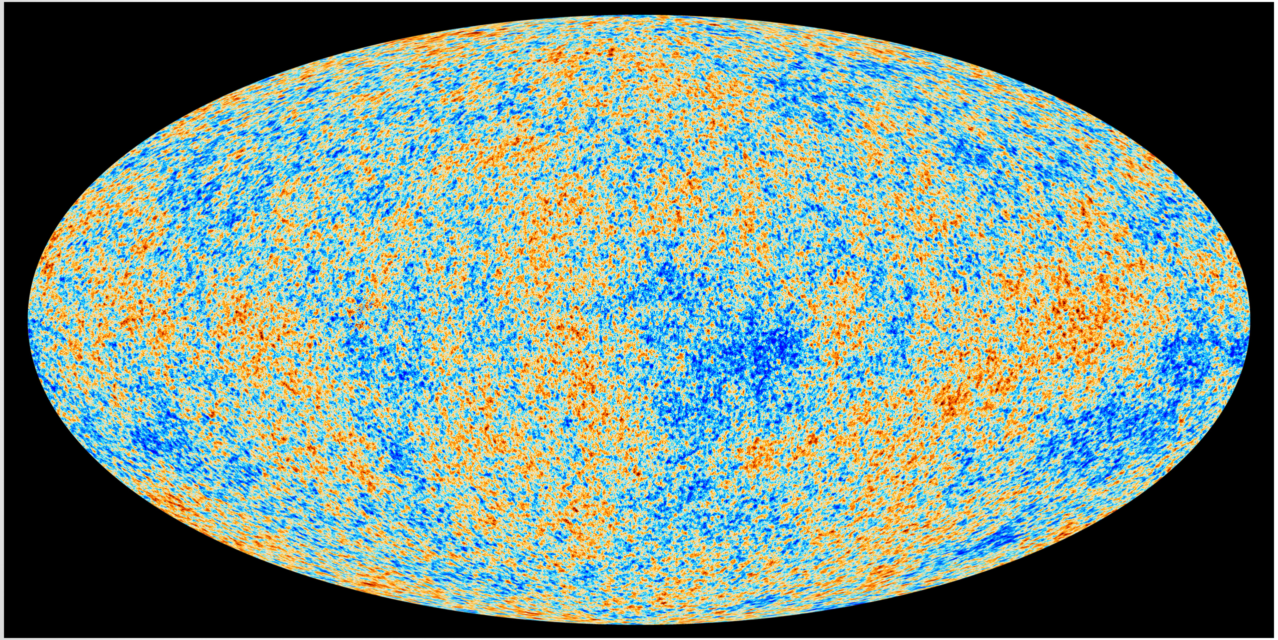
WMAP image of the *Cosmic Microwave Background Radiation*



http://www.nasa.gov/centers/goddard/images/content/96115main_Full_m.jpg

- WMAP satellite *launched* 06/01, *ended* 10/10
- WMAP image from 7 years of data!
- Data implies that universe is *flat*.

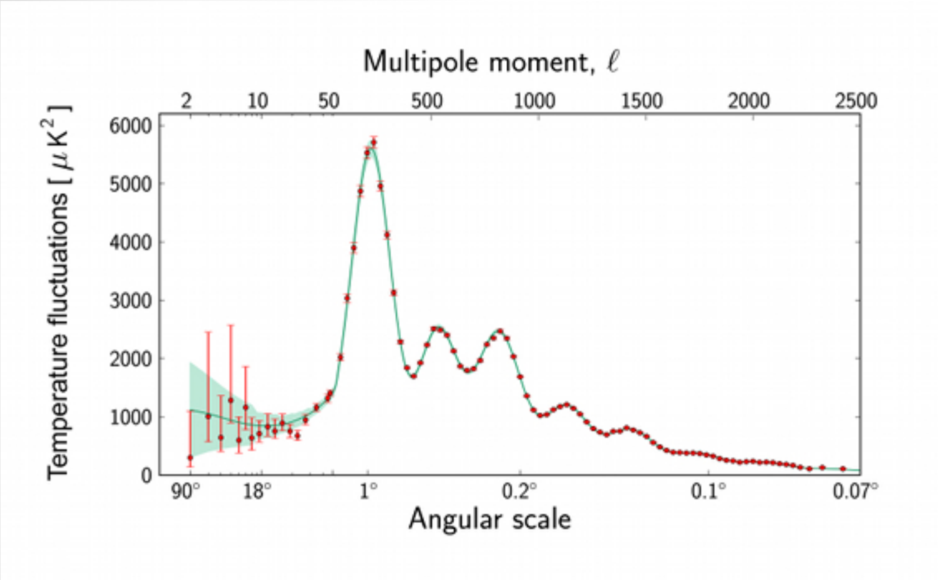
Planck image of the *Cosmic Microwave Background Radiation*



https://www.nasa.gov/mission_pages/planck/#.Vvs8mnDgI2s

- Planck satellite *launched* 05/09, *ended* 10/13
- 2.5 x *greater* resolution than WMAP (1/12°)
- Measured *polarization* of light from early universe
- Found strong evidence for *inflation*.

Detailed analysis of *temperature variations* from Planck & WMAP data



- Green curve is line of best fit.

$$\Omega_{r,0} \sim 10^{-4}$$
$$\Omega_{m,0} = 0.309$$
$$\Omega_{v,0} = 0.691$$

$$\Omega_{m,0} \rightarrow \Omega_{\text{baryons}} = 0.049$$
$$\Omega_{m,0} \rightarrow \Omega_{\text{DM}} = 0.259$$

All ordinary matter (stars, galaxies, etc..) comprises only 4.9% of the total matter/energy of the Universe!

Inflationary Theory

Consider a *scalar-field* coupled to gravity

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

where

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi + \left[\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi - V(\phi) \right] g_{\mu\nu}$$

and where $V(\phi)$ describes the *self-interaction* of the scalar field

Georges Lemaître suggests the Universe had a beginning..

- Belgian Astrophysicist/Catholic Priest

- 1927 paper in *Annals of the Brussels Scientific Society**

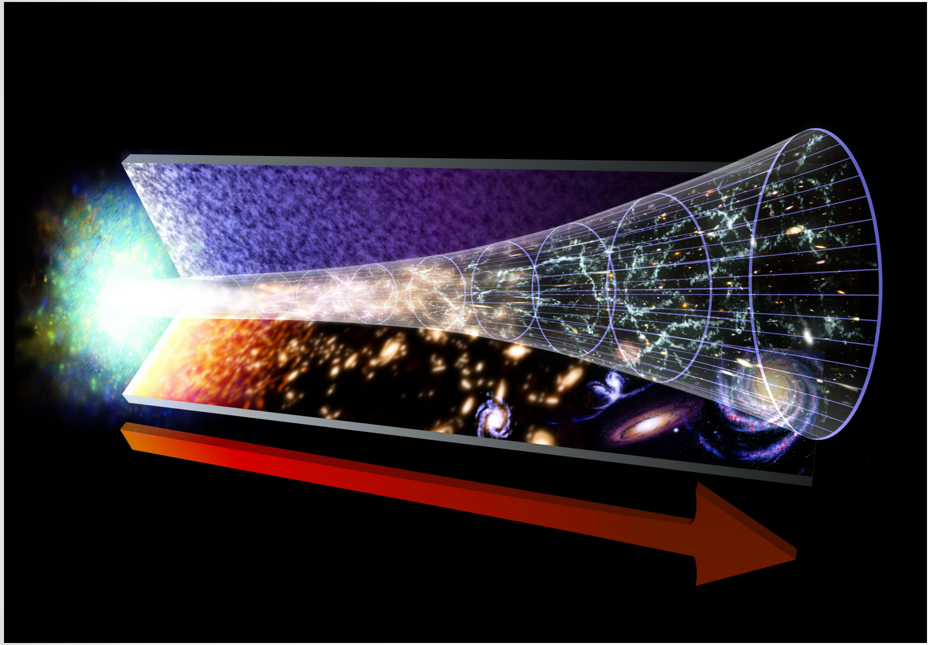
- showed that the universe had to be either *contracting* or *expanding*.

- suggested that the Universe had a definite *beginning* in which all its matter & energy were concentrated @ *one point*.

http://www.aquinasandmore.com/resources/georges-lemaitres.jpg

* Lemaître, G. Ann. Soc. Sci. Brux. A 47, 49–59 (1927).

Did the Universe begin with a *“Big Bang”*?



https://svs.gsfc.nasa.gov/vis/a010000/a010100/a010128/Arrow_JPG.jpg

The *Big Bang*…

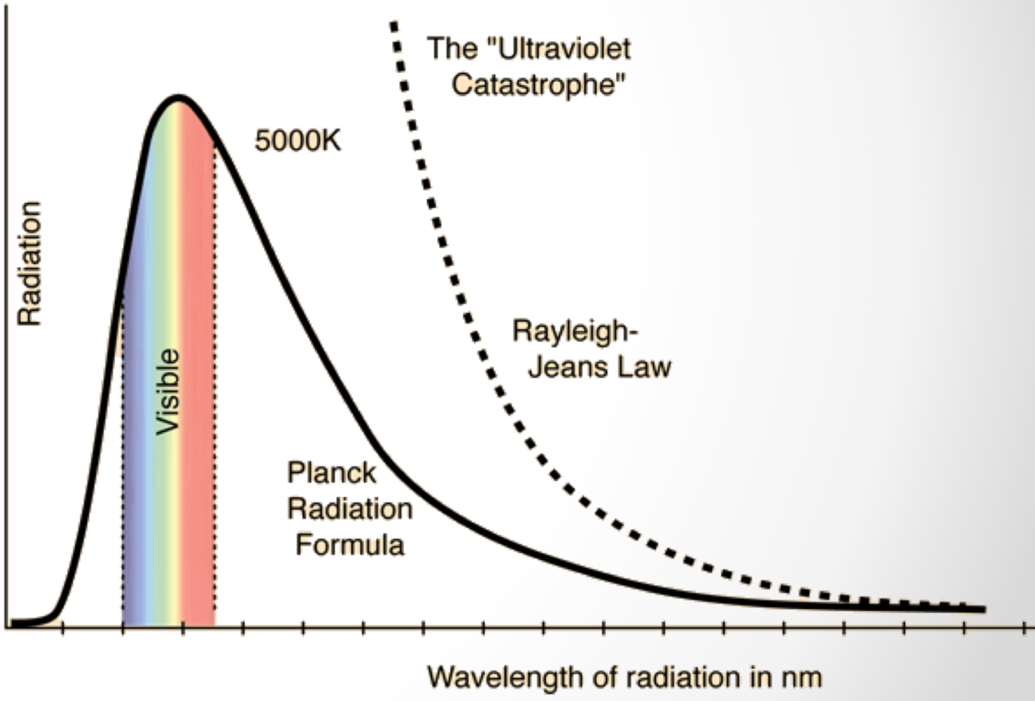
- is ***not*** an explosion that happened @ ***one*** pt in *space*
- occurred at ***all*** pts in *space* @ ***one*** moment in *time*

***Big Bang* -** a **time** of *infinite* density, *infinite* temperature, and *infinite* spacetime curvature

In the early 1960s, the Princeton group in gravitational physics…

- finds that the Universe should be uniformly bathed in a background *microwave radiation*

- predicts a *blackbody spectrum* of the background radiation with $T \sim 10$K



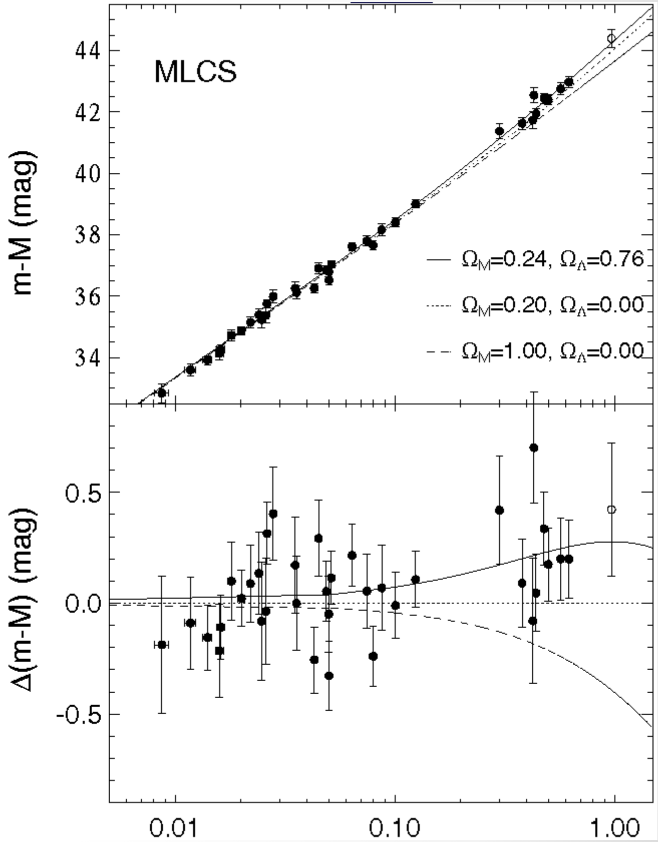
http://hyperphysics.phy-astr.gsu.edu/hbase/imgmod/uvcatas.gif

Data from Type Ia Supernovae, WMAP and SDSS implies…

- The expansion of the Universe is *ACCELERATING!*
- *seems* to indicate a *vacuum energy*

Saul Perlmutter, Brian Schmidt, & Adam Riess

Awarded the 2011 Nobel Prize in physics *for the discovery of the accelerating expansion of the Universe through observations of distant supernovae.*



http://nedwww.ipac.caltech.edu/level5/Carroll2/Figures/figure3.jpeg
High-z Supernova Search Team: Riess et al., 1998