A possible higher-dimensional alternative to scalar-field inflationary theory[1]

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[1] *submitted to Phys. Rev. D*

Table of Contents

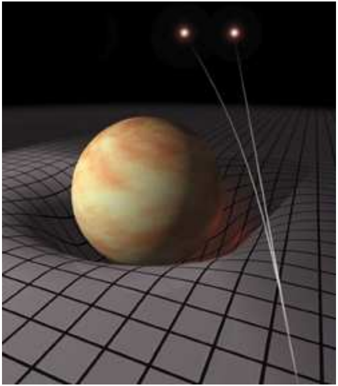
Table of Contents

Anisotropic D-dimensional FRW cosmology

The *Einstein field equations* in D spacetime dimensions are

$$G_A{}^B = \frac{8\pi G_D}{c^2} T_A{}^B$$

- $G_A{}^B$ describes the *curvature* of spacetime
- $T_A{}^B$ describes the *matter and energy* in spacetime



Anisotropic D-dimensional FRW cosmology

We choose a *metric ansatz* of the form

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] + b^2(t)(dy_1^2 + dy_2^2 + ... + d_d^2)$$

The *stress-energy tensor* is assumed to be that of a *perfect fluid*

$$T_A{}^B = \mathrm{diag}\left[-\rho(t), p(t), p(t), p(t), p_d(t), ..., p_d(t)\right]$$

where $p(t)$ and $p_d(t)$ are the pressures of the 3D and higher-dimensional spaces.

Anisotropic D-dimensional FRW cosmology

The $D = d + 4$ dimensional Friedmann-Robertson-Walker (FRW) field equations and the conservation equation are

$$\begin{aligned}
\rho &= 3\frac{\dot{a}^2}{a^2} + \frac{1}{2}d(d-1)\frac{\dot{b}^2}{b^2} + 3d\frac{\dot{a}\dot{b}}{ab} \\
p &= -\left[2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + d\frac{\ddot{b}}{b} + \frac{1}{2}d(d-1)\frac{\dot{b}^2}{b^2} + 2d\frac{\dot{a}\dot{b}}{ab}\right] \\
p_d &= -\left[3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) + (d-1)\left(\frac{\ddot{b}}{b} + \frac{1}{2}(d-2)\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}\dot{b}}{ab}\right)\right] \\
0 &= \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) + d\frac{\dot{b}}{b}(\rho + p_d)
\end{aligned}$$

Anisotropic D-dimensional FRW cosmology

By adopting two equations of state of the form

$$p = w\ \rho$$
$$p_d = v\ \rho,$$

we obtain the *exact* differential equation of the form

$$\frac{d}{dt}\left[a^{3-dn}\ \frac{d}{dt}\left(a^n b\right)^d\right] = 0$$

Anisotropic D-dimensional FRW cosmology

The higher-dimensional scale factor takes the form

$$b(t) = \frac{1}{a^n(t)} \left[\gamma_1 + \gamma_0 \int a(t)^{(dn-3)} dt \right]^{1/d}$$

where γ_1 and γ_0 are constants of integration.

We defined the power

$$n \equiv \left[\frac{2 - 3\varepsilon}{d - (d-1)\varepsilon} \right]$$

where

$$\varepsilon \equiv \frac{(1-w)}{(1-v)}$$

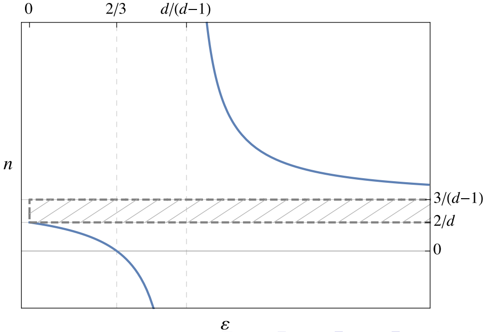


Table of Contents

Effective 4D FRW field equations

The FRW field equations can be written in the form

$$
\begin{aligned}
\rho &= \eta_1 \frac{\dot{a}^2}{a^2} - \frac{1}{dn}(2\eta_1 + 3\eta_2)\frac{\gamma_0}{x}\frac{\dot{a}}{a} + \frac{1}{2d}(d-1)\frac{\gamma_0^2}{x^2} \\
\tilde{p} &= -\frac{1}{3}\eta_1 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \left[\frac{1}{dn}(2\eta_1 + 3\eta_2) - \frac{2}{3}(\eta_1 + 3\eta_2)\right]\frac{\gamma_0}{x}\frac{\dot{a}}{a} \\
&\quad -\frac{1}{3}(1 + 2\eta_2)\cdot\frac{1}{2d}(d-1)\frac{\gamma_0^2}{x^2} \\
p_d &= \frac{1}{dn}\left[(2\eta_1 + 3\eta_2)\frac{\ddot{a}}{a} + (2\eta_1 - \eta_2(\eta_1 + 3\eta_2))\frac{\dot{a}^2}{a^2}\right] + \frac{1}{2d}(d-1)\frac{\gamma_0^2}{x^2} \\
0 &= \dot{\rho} + 3(1 + \tilde{w})\frac{\dot{a}}{a}\rho + (1 + v)\frac{\gamma_0}{x}\rho
\end{aligned}
$$

where we defined the *higher-dimensional volume element*, $x(t)$, as

$$
x(t) \equiv a^3 b^d = a(t)^{(3-dn)}\left[\gamma_1 + \gamma_0 \int a(t)^{(dn-3)}dt\right]
$$

Effective 4D FRW field equations

The FRW field equations can be written in the form

$$\begin{aligned}
\rho &= \qquad\qquad [] \frac{\dot{a}^2}{a^2} + [] \frac{\gamma_0}{x}\frac{\dot{a}}{a} + [] \frac{\gamma_0^2}{x^2} \\
\tilde{w}\rho &= [] \frac{\ddot{a}}{a} + [] \frac{\dot{a}^2}{a^2} + [] \frac{\gamma_0}{x}\frac{\dot{a}}{a} + [] \frac{\gamma_0^2}{x^2} \\
v\rho &= [] \frac{\ddot{a}}{a} + [] \frac{\dot{a}^2}{a^2} \qquad\qquad + [] \frac{\gamma_0^2}{x^2}
\end{aligned}$$

where

$$x(t) = \frac{1}{a(t)^{(dn-3)}} \left[\gamma_1 + \gamma_0 \int a(t)^{(dn-3)} dt\right] \equiv \gamma_0 \frac{f}{\dot{f}}$$

Effective 4D FRW field equations

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- Remarkably, these equations can be solved *exactly*!

Table of Contents

General treatment of the effective 4D FRW field equations

The general solution is

$$f^{1/\beta_\pm} \cdot {}_2F_1\left(\frac{\alpha}{(1+\alpha)}, -\frac{1}{\beta_\pm\delta_\pm}; 1-\frac{1}{\beta_\pm\delta_\pm}; \left(\frac{\gamma_1}{f}\right)^{\delta_\pm}\right) = \frac{1}{\beta_\pm}\left(c_0(1+\alpha)\frac{\gamma_0^{1/\alpha}}{\delta_\pm\gamma_1^{\delta_\pm}}\right)^{\alpha/(1+\alpha)} (t-t_0)$$

$$\text{where} \quad f \equiv \gamma_1 + \gamma_0 \int a^{(dn-3)} dt$$

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- General solution yields $t = t(a)$ & *can't* be inverted to yield $a = a(t)$.

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- General solution yields $t = t(a)$ & *can't* be inverted to yield $a = a(t)$.
- Hypergeometric functions, ${}_2F_1(a, b; c; z)$, have singularities at $z = 0, 1$, and ∞, which can be expanded about to yield *approximate solutions*.

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- Hypergeometric functions, ${}_2F_1(a, b; c; z)$, have singularities at $z = 0, 1$, and ∞, which can be expanded about to yield *approximate solutions*.
- These approximate solutions equate to the perturbative solutions of the *fluid* (near $z \sim 1$) & *volume* regimes (near $z \sim 0$ & $z \sim \infty$).

Table of Contents

The fluid regime

Here we are interested in the approximate solution *when*

$$\rho \gg \frac{\gamma_0}{x}\frac{\dot{a}}{a} \gg \frac{\gamma_0^2}{x^2} \quad \text{as} \quad \frac{\dot{a}}{a} \gg \frac{\gamma_0}{x} \quad \text{and} \quad \gamma_1 \gg \gamma_0 \int a(t)^{(dn-3)} dt$$

In this regime, the field equations take the form

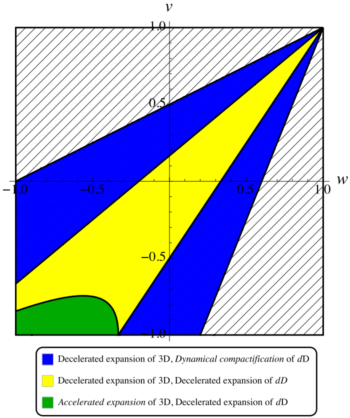
$$\begin{aligned}
\rho &= \eta_1 \frac{\dot{a}^2}{a^2} + [] \frac{\gamma_0}{x}\frac{\dot{a}}{a} + [] \frac{\gamma_0^2}{x^2} \\
\tilde{p} &= -\frac{1}{3}\eta_1 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) + [] \frac{\gamma_0}{x}\frac{\dot{a}}{a} + [] \frac{\gamma_0^2}{x^2} \\
p_d &= [] \frac{\ddot{a}}{a} + [] \frac{\dot{a}^2}{a^2} + [] \frac{\gamma_0^2}{x^2}
\end{aligned}$$

The fluid regime

To lowest order, the 3D and higher-dimensional scale factors take the form

$$a(t) = \tilde{a}_0 t^{2/3(1+\tilde{w})}$$
$$b(t) = \tilde{b}_0 t^{-2n/3(1+\tilde{w})} \quad \text{for } \tilde{w} \neq -1$$

$$\text{where} \quad \tilde{b}_0 \equiv \frac{\gamma_1^{1/d}}{\tilde{a}_0^n}$$



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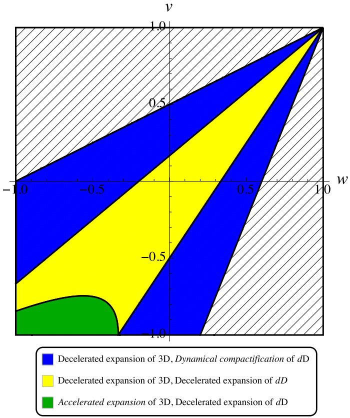

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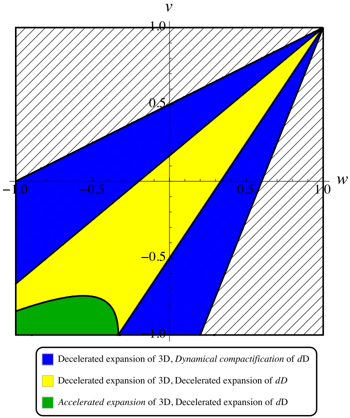

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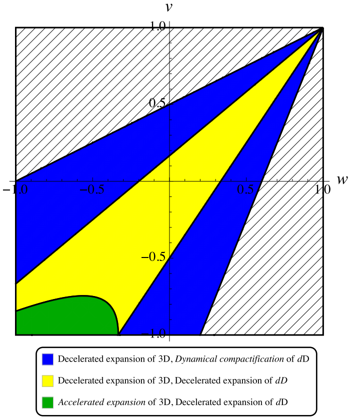

The fluid regime solution...

- closely mimics that of standard 4D FRW cosmology.
- gives rise to a *late-time accelerated expansion* driven by vacuum energy.
- becomes valid in the early universe following an even earlier *volume regime* solution.

Table of Contents

The volume regime

Here we are interested in the approximate solution in the regime[2] *when*

$$\rho \ll \frac{\gamma_0}{x}\frac{\dot{a}}{a} \sim \frac{\gamma_0^2}{x^2} \quad \text{as} \quad \frac{\dot{a}}{a} \sim \frac{\gamma_0}{x} \quad \text{and} \quad \gamma_1 \ll \gamma_0 \int a(t)^{(dn-3)} dt$$

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$$\begin{aligned} \rho &= \eta_1 \frac{\dot{a}^2}{a^2} + [\,]\,\frac{\gamma_0}{x}\frac{\dot{a}}{a} + [\,]\,\frac{\gamma_0^2}{x^2} \\ \tilde{p} &= -\frac{1}{3}\eta_1 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) + [\,]\,\frac{\gamma_0}{x}\frac{\dot{a}}{a} + [\,]\,\frac{\gamma_0^2}{x^2} \\ p_d &= [\,]\,\frac{\ddot{a}}{a} + [\,]\,\frac{\dot{a}^2}{a^2} + [\,]\,\frac{\gamma_0^2}{x^2} \end{aligned}$$

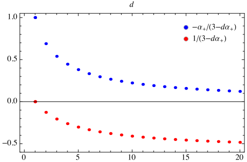
[2] with the *unique exception* of one of two solutions for $d = 1$.

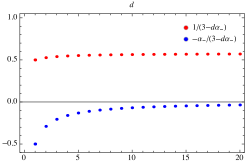
The volume regime

To lowest order, the 3D and higher-dimensional scale factors take the form

$$a(t) = a_0 t^{1/(3-d\alpha_\pm)}$$
$$b(t) = b_0 t^{-\alpha_\pm/(3-d\alpha_\pm)}$$

where $b_0 \equiv \left(\frac{\gamma_0}{\beta_\pm a_0^3}\right)^{1/d}$





The volume regime solutions...

[3] Phys. Rev. D **21**, 2167 (1980)

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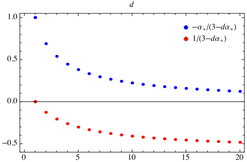

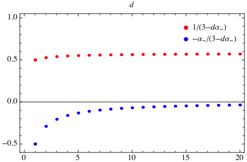

The volume regime solutions...

- generalize the D-dimensional vacuum solutions of Chodos and Detweiler[3]

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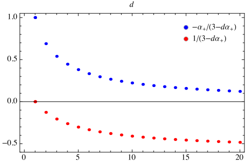

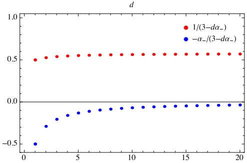

The volume regime solutions...

- generalize the D-dimensional vacuum solutions of Chodos and Detweiler[3]
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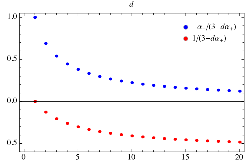

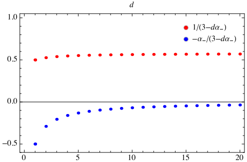

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The uniquely special case of the 5D solution

For $d = 1$, the regime of validity is dictated by the strong inequalities

$$\frac{\dot{a}^2}{a^2} \ll \rho \sim \frac{\gamma_0}{x}\frac{\dot{a}}{a} \quad \text{as} \quad \frac{\dot{a}}{a} \ll \frac{\gamma_0}{x} \quad \text{and} \quad \gamma_1 \ll \gamma_0 \int a(t)^{(n-3)} dt$$

In this regime, the field equations take the form

$$\begin{aligned} \rho &= \eta_1 \frac{\dot{a}^2}{a^2} + 3\frac{\gamma_0}{x}\frac{\dot{a}}{a} \\ \tilde{p} &= -\frac{1}{3}\eta_1 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) + \frac{\gamma_0}{x}\frac{\dot{a}}{a} \\ p_d &= -3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \end{aligned}$$

The uniquely special case of the 5D solution

Keeping the first-order correction term, the 3D scale factor takes the form

$$a(t) = a_0 \left[1 + \kappa t^{(1-v)}\right]$$

where the *Hubble parameter* and *acceleration* take the form

$$\begin{aligned} \frac{\dot{a}}{a} &= \frac{\rho_0 a_0^{3(v-w)}}{3\gamma_0^{(1+v)}} t^{-v} \\ \frac{\ddot{a}}{a} &= -v \cdot \frac{\rho_0 a_0^{3(v-w)}}{3\gamma_0^{(1+v)}} t^{-(1+v)} \end{aligned}$$

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- features a *Hubble parameter* that increases from an initial value of zero and drives the earliest expansion of the 3D scale factor from an initial *constant* value for $v < 0$.

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This $d = 1$ volume regime solution...

- features a *Hubble parameter* that increases from an initial value of zero and drives the earliest expansion of the 3D scale factor from an initial *constant* value for $v < 0$.
- exhibits *accelerated expansion* for *any* 3D EoS parameter, w, for $v < 0$, which diverges for vanishing small time.

Table of Contents

A possible 5D alternative to scalar-field inflationary theory

Inserting the perturbative solutions into the strong inequalities

$$1 \gg \frac{t_{\text{vol},i}}{t} \tag{1}$$

$$1 \gg \left(\frac{t_{\text{vol},i}}{t}\right)^{(1+v)} \left(\frac{\tilde{a}_0\, t^{2/3(1+\tilde{w})}}{a_0}\right)^{3(1+\tilde{w})} \tag{2}$$

$$1 \ll \left(\frac{t_{\text{vol},i}}{t}\right) \left(\frac{\tilde{a}_0\, t^{2/3(1+\tilde{w})}}{a_0}\right)^{(3-n)} \tag{3}$$

where

$$t_{\text{vol},i} \equiv \frac{\gamma_1}{\gamma_0} a_0^{3-n}$$

Now...

A possible 5D alternative to scalar-field inflationary theory

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$$t_{\mathrm{vol},i} \equiv \frac{\gamma_1}{\gamma_0} a_0^{3-n}$$

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- when (1) *becomes* satisfied, the *volume regime* solution is 'turned on'.

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A possible 5D alternative to scalar-field inflationary theory

Inserting the perturbative solutions into the strong inequalities

$$1 \gg \frac{t_{\mathrm{vol},i}}{t} \tag{1}$$

$$1 \gg \left(\frac{t_{\mathrm{vol},i}}{t}\right)^{(1+v)} \left(\frac{\tilde{a}_0 \, t^{2/3(1+\tilde{w})}}{a_0}\right)^{3(1+\tilde{w})} \tag{2}$$

$$1 \ll \left(\frac{t_{\mathrm{vol},i}}{t}\right) \left(\frac{\tilde{a}_0 \, t^{2/3(1+\tilde{w})}}{a_0}\right)^{(3-n)} \tag{3}$$

where

$$t_{\mathrm{vol},i} \equiv \frac{\gamma_1}{\gamma_0} a_0^{3-n}$$

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- when (1) *becomes* satisfied, the *volume regime* solution is 'turned on'.
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- when (3) *becomes* satisfied, the *fluid regime* solution is 'turned on'.

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- parameterize $a_0 = 10^m$, where we expect $m \sim -30, -31$.

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Equations (2) and (3) can be written as

$$1 \gg \left(\frac{t}{t_{\text{vol},f}}\right)^{(1-v)} \quad \text{where} \quad t_{\text{vol},f} \equiv 10^q \text{ s}$$

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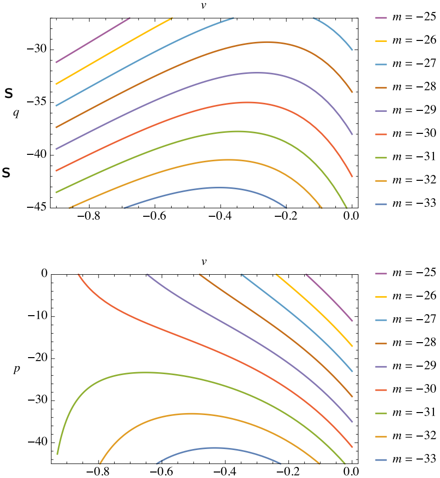

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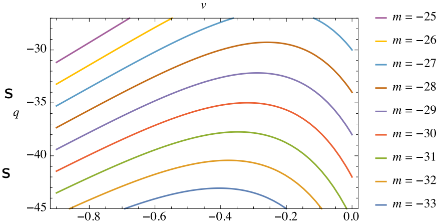

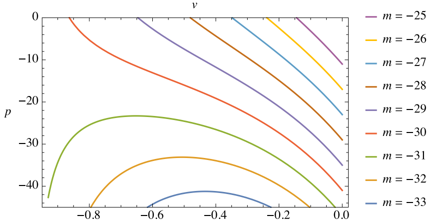

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- For $m < -33$, the volume regime solution turns off *before* the Planck time.

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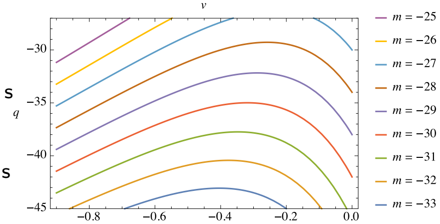

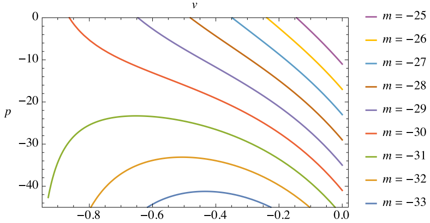

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- For $m < -33$, the volume regime solution turns off *before* the Planck time.
- For $m > -25$ the fluid regime solution doesn't turn on until *after* $t = 1$s.

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- The time scales describing these phenomena fall within a *tiny fraction of 1s*, when the fundamental constants of this theory are aligned with the Planck time.