Physics 311

Homework Set 8

Due Thursday, October 13

1. Consider four point charges located at the corners of a square with charges and coordinates

$$\begin{aligned} q_1 = q \ \ @ \ \ x = y = 0 \qquad &, \qquad q_2 = 2q \ \ @ \ \ x = \ell \ , \ y = 0 \\ q_3 = -q \ \ @ \ \ x = y = \ell \qquad &, \qquad q_4 = 3q \ \ @ \ \ x = 0 \ , \ y = \ell . \end{aligned} \tag{1}$$

   a) Calculate the electric potential of charges q_2, q_3, and q_4 at the origin of the coordinate system.

   b) How much work was required to bring in charge q_1 from infinity and place it at the origin?

   c) Calculate the work necessary to assemble the whole configuration of four charges.

   Answer:

$$W = \frac{5\sqrt{2}}{2} \cdot \frac{1}{4\pi\epsilon_0} \frac{q^2}{\ell} \tag{2}$$

2. Consider a uniform sphere of charge density ρ_0, radius R, and charge $+Q$.

   a) Considering a concentric spherical Gaussian surface of radius $r < R$, calculate the total charge enclosed. Write this in terms of the total charge.

   b) Using Gauss' Law in integral form, find the $\vec{E}$-field of the charged sphere for $r < R$.

   c) Using the fact that the electrostatic potential can be calculated via

$$V(\vec{r}\,) = -\int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot d\vec{\ell}, \tag{3}$$

   find $V(r < R)$ using infinity as your reference point, $\mathcal{O}$.

   Answer:

$$V(r < R) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{2R} \left(3 - \frac{r^2}{R^2} \right) \tag{4}$$

3. Reconsider a uniform sphere of charge density ρ_0, radius R, and charge Q.

   a) Using the fact that the work necessary to assemble the charged sphere can be calculated via

   $$W = \frac{1}{2}\int \rho V \, d\tau, \tag{5}$$

   calculate the energy stored in the sphere.

   b) When calculating part a), what are the limits of integration for the radial integral?

   c) Using the fact that the work necessary to assemble the charged sphere can be calculated via

   $$W = \frac{\epsilon_0}{2}\int E^2 d\tau, \tag{6}$$

   calculate the energy stored in the sphere.

   d) When calculating part c), what are the limits of integration for the radial integral?

   e) Using the fact that the work necessary to assemble the charged sphere can be calculated via

   $$W = \frac{\epsilon_0}{2}\left(\int_{\mathcal{V}} E^2 d\tau + \oint_{\mathcal{S}} V\vec{E}\cdot d\vec{a}\right), \tag{7}$$

   calculate the energy stored in the sphere. When performing this calculation, consider a concentric sphere of radius $a > R$.

4. Consider a *metal sphere* of radius R, which carries a net charge of $+Q$.

   This metal sphere is surrounded by a *concentric metal shell* with an inner radius a and outer radius b. The metal shell has *zero* net charge.

   a) Find the surface charge density, $\sigma(r)$, at $r = R$, at $r = a$, and at $r = b$.[1]

   b) Find the $\vec{E}$-field in each of the regions.[2]

   c) Using the $\vec{E}$-field in each region, calculate the potential at the center of the metal sphere using infinity as your reference point, $\mathcal{O}$.

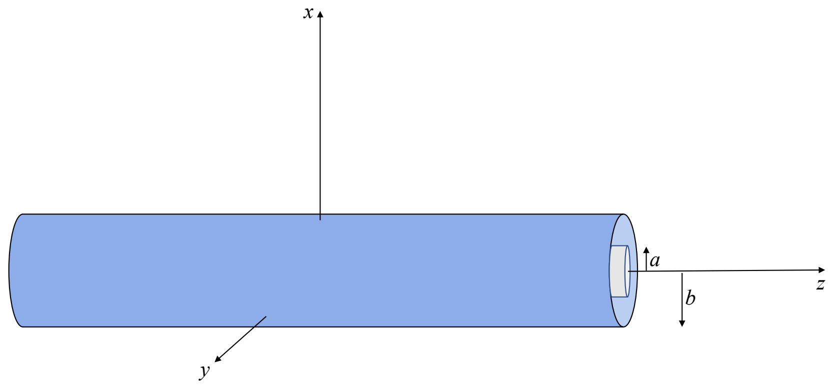
   d) Now consider touching the outer surface to a grounding wire.

   How do the answers to parts a), b), and c) change?

[1] Hint: Think Gauss's Law in integral form, concentric Gaussian surfaces in each region, and Q_{enc}.

[2] Hint: Think Gauss's Law in integral form and concentric Gaussian surfaces in each region.

5. Consider a long, metal coaxial cable with inner radius a and outer radius b, as shown in the figure below. In a length ℓ of cable, the inner surface has charge $+Q$ and the outer surface has charge $-Q$, so that a given length of cable has zero net charge.



a) Find the surface charge density, $\sigma(s)$, at $s = a$ and $s = b$. Call these σ_a and σ_b.

b) Find $\sigma_b(\sigma_a)$. Which has a larger magnitude, $|\sigma_a|$ and $|\sigma_b|$? Are they the same?

c) Find the $\vec{E}$-field in each of the three regions.

d) Calculate the potential difference between the two surfaces.

e) Calculate the capacitance.

Answer:

$$C = \frac{2\pi\epsilon_0\ell}{\ln(b/a)} \tag{8}$$