Physics 311

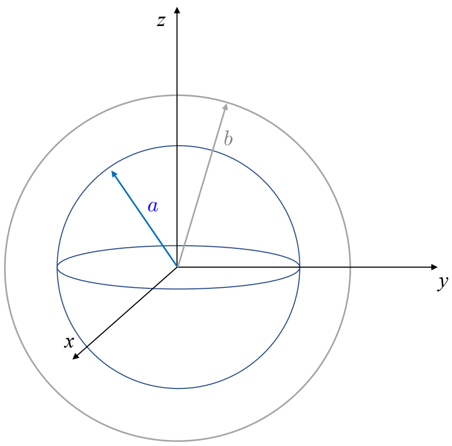
Homework Set 7

Due Monday, October 3

1. Reconsider the *hollow spherical shell* with a spherically-symmetric charge distribution

$$\rho(r) = kr^{\pi} \quad \text{for} \quad a \leq r \leq b \tag{1}$$

for some constant k.



In Homework Set 6, you considered this charge density, computed the $\vec{E}$- field in the three distinct regions, and found the results

$$\begin{aligned} \vec{E}(r) &= 0 & \text{for} \quad r < a \\ \vec{E}(r) &= \frac{k}{(\pi+3)\epsilon_0}\left(r^{\pi+1} - \frac{a^{\pi+3}}{r^2}\right)\hat{r} & \text{for} \quad a < r < b \\ \vec{E}(r) &= \frac{k}{(\pi+3)\epsilon_0}\frac{(b^{\pi+3} - a^{\pi+3})}{r^2}\,\hat{r} & \text{for} \quad b < r. \end{aligned} \tag{2}$$

An electrostatic potential describing an $\vec{E}$-field can be found by calculating the line integral of the vector function via

$$V(\vec{r}\,) = -\int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot d\vec{\ell}. \tag{3}$$

Using the electric field in the three regions, calculate the potential at the center of the spherical shell, using infinity as your reference point, $\mathcal{O}$. In calculating the electrostatic potential, use an integration path which points purely in the radial direction.

Answer:

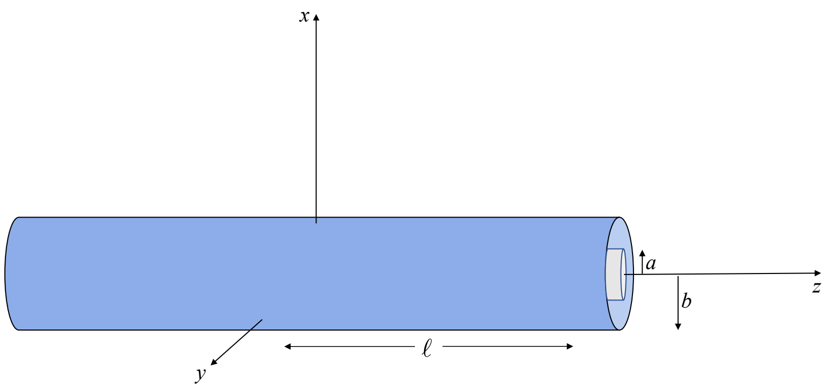
$$V(r=0) = \frac{k}{(\pi+2)\epsilon_0}\left(b^{\pi+2} - a^{\pi+2}\right) \tag{4}$$

2. Reconsider the long coaxial cable with a *volume* charge density

$$\rho(s) = \rho_0 \frac{s}{a} \quad \text{for} \quad s \leq a \tag{5}$$

in the inner cylinder and a *uniform surface* charge density on the outer cylindrical shell

$$\sigma(s) = \sigma_0 \quad \text{at} \quad s = b, \tag{6}$$

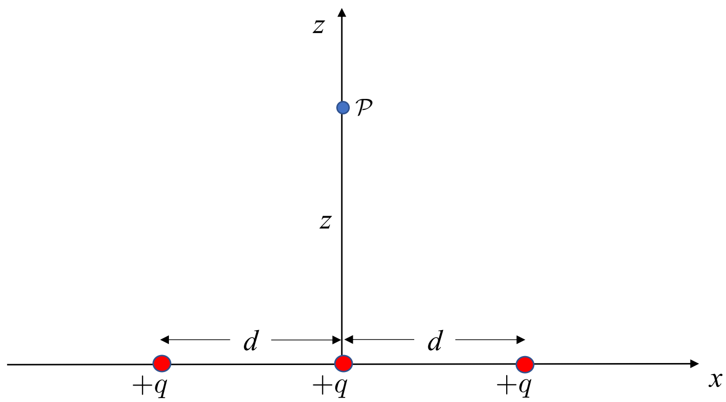


where the surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral. In Homework Set 6, you considered this charge density, computed the $\vec{E}$- field in the three distinct regions, and found the results

$$\begin{aligned} \vec{E}(s) &= \frac{\rho}{3\epsilon_0}\frac{s^2}{a}\hat{s} && \text{for} \quad s < a \\ \vec{E}(s) &= \frac{\rho}{3\epsilon_0}\frac{a^2}{s}\hat{s} && \text{for} \quad a < s < b \\ \vec{E}(s) &= 0 && \text{for} \quad b < s. \end{aligned} \tag{7}$$

Using the electric field in the three regions, calculate the potential difference between a point on the axis and a point on the outer cylinder.

3. Consider three equal point charges, each with charge $+q$, lying on the x-axis with one charge at the origin and the other two a distance d away, as shown in the figure below.



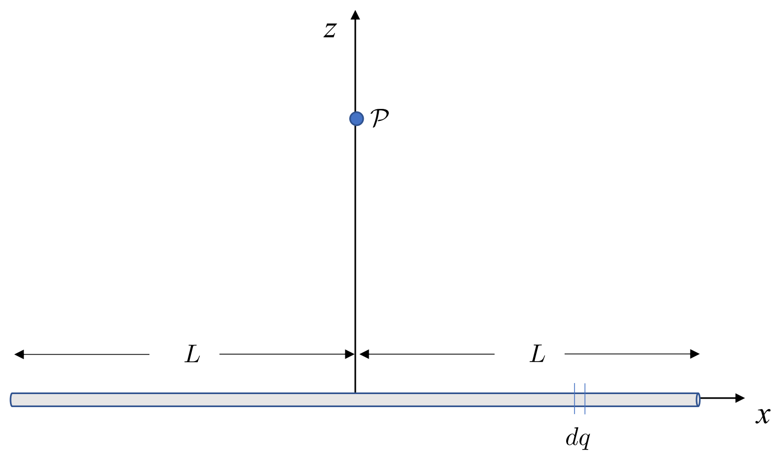
a) Calculate the *electric potential* of the three charges at point $\mathcal{P}$ via

$$V(\vec{r}\,) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{\mathscr{r}_i}. \tag{8}$$

b) Calculate the *electric field* of the three charges at point $\mathcal{P}$ via

$$\vec{E} = -\vec{\nabla} V. \tag{9}$$

4. Consider a line charge of length $2L$ and uniform charge density, $\lambda(x) = \lambda_0$, lying on the x-axis with its center at the origin as shown in the figure below. In this problem, we wish to construct the electric potential, $V(z)$, and the $\vec{E}$-field of this line charge at point $\mathcal{P}$.



a) On the diagram, *draw* the position vector $\vec{r}$ of the *field point*, $\mathcal{P}$, the position vector $\vec{r}\,'$ of the *source point*, dq, and the separation vector $\vec{\mathscr{r}}$ from the source point to the field point.

b) In unit-vector notation, construct the separation vector, $\vec{\mathscr{r}}$.

c) Construct the magnitude of the separation vector, $\mathscr{r}$.

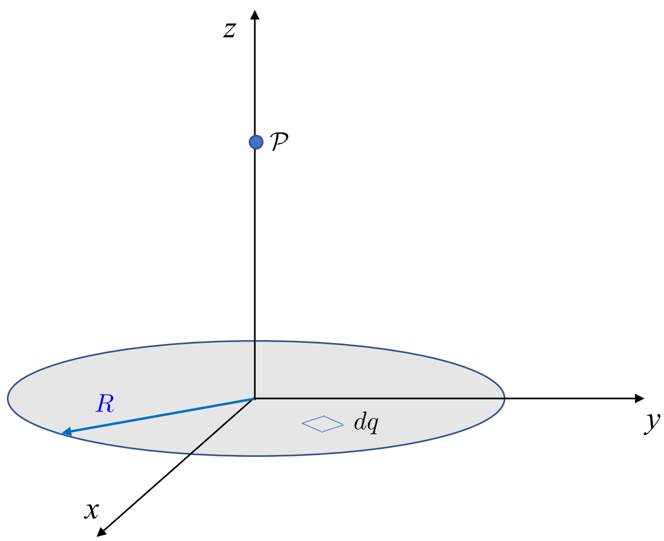
d) Calculate the *electric potential* of the line charge at point $\mathcal{P}$ via

$$V(\vec{r}\,) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}\,')}{\mathscr{r}} d\ell'. \tag{10}$$

e) Calculate the *electric field* of the line charge at point $\mathcal{P}$ via

$$\vec{E} = -\vec{\nabla} V. \tag{11}$$

5. Reconsider a charged, flat circular disk of radius R lying on the xy-plane with its center at the origin with a uniform surface charge density, $\sigma(s) = \sigma_0$, as shown in the figure below. In this problem, we ultimately wish to construct the electric potential, $V(z)$, and the $\vec{E}$-field of this charged disk at point $\mathcal{P}$.



a) Using cylindrical coordinates, construct the magnitude of the separation vector, $\mathscr{r}$, and the area element, da'.

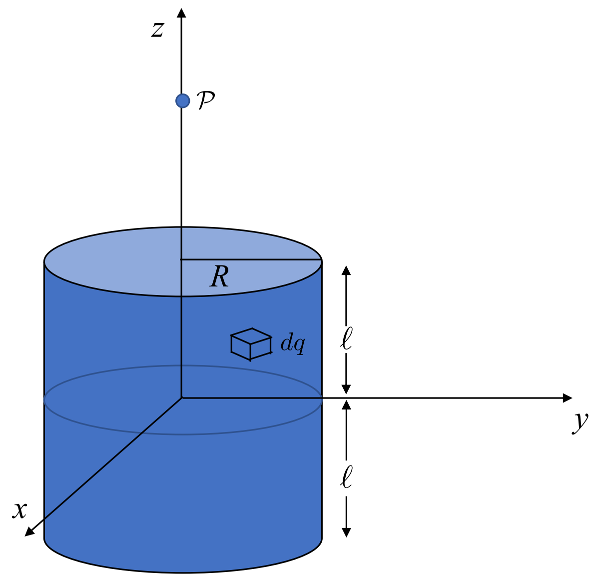
b) Calculate the *electric potential* of the charged disk at point $\mathcal{P}$ via

$$V(\vec{r}\,) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r}\,')}{\mathscr{r}}da'. \tag{12}$$

c) Calculate the *electric field* of the charged disk at point $\mathcal{P}$ via

$$\vec{E} = -\vec{\nabla}V. \tag{13}$$

6. Consider a uniformly charged solid cylinder of density ρ_0 of radius R and height 2ℓ with its center at the origin, as shown in the figure below. In this problem, we ultimately wish to construct the electric potential , $V(z)$, and the $\vec{E}$-field of this cylinder at point $\mathcal{P}$.



a) On the diagram, *draw* the position vector $\vec{r}$ of the *field point*, $\mathcal{P}$, the position vector $\vec{r}\,'$ of the *source point*, dq, and the separation vector $\vec{\mathscr{r}}$ from the source point to the field point.

b) Using cylindrical coordinates, construct the position vectors, $\vec{r}$ and $\vec{r}\,'$, the separation vector, $\vec{\mathscr{r}}$, and the magnitude of the separation vector, $\mathscr{r}$

c) Calculate the *electric potential* of the cylinder at point $\mathcal{P}$ via

$$V(\vec{r}\,) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}} d\tau'. \tag{14}$$

Answer:

$$V(z) = \frac{\rho_0}{4\epsilon_0}\left[(z+\ell)\sqrt{(z+\ell)^2+R^2} - (z-\ell)\sqrt{(z-\ell)^2+R^2} - 4\ell z + R^2 \ln\left(\frac{\sqrt{(z+\ell)^2+R^2}+(z+\ell)}{\sqrt{(z-\ell)^2+R^2}+(z-\ell)}\right)\right] \tag{15}$$