Physics 311

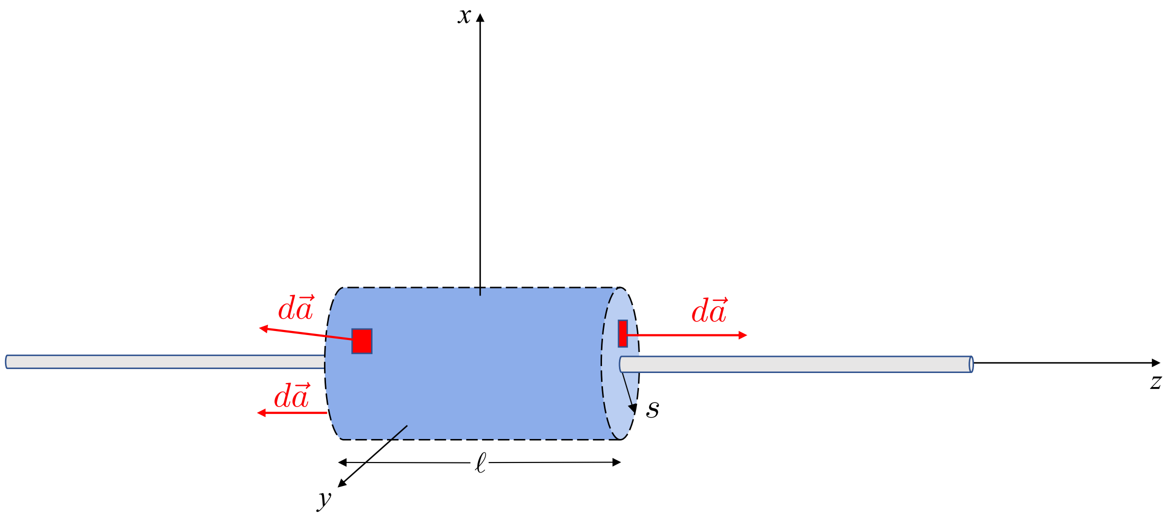
Homework Set 6

Due Monday, September 26

1. Consider an *infinitely long straight wire* with a uniform line charge

$$\lambda(z) = \lambda_0, \tag{1}$$

and the cylindrical, coaxially Gaussian surface of radius s and length ℓ, as shown in the figure below.



a) Explicitly calculate the total charge enclosed within the Gaussian surface, Q_{enc}.

b) Using cylindrical coordinates, construct the differential area element, $d\vec{a}$, for *each* of the three sides of the cylindrical Gaussian surface.

c) Explicitly calculate the flux of the $\vec{E}$-field through each of the three surfaces. Sum the three contributions to yield the total flux through the closed surface,

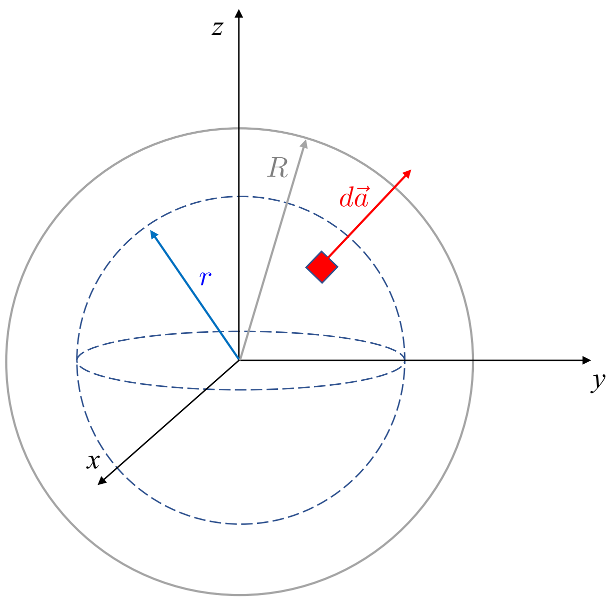
$$\oint \vec{E} \cdot d\vec{a}. \tag{2}$$

d) Using Gauss' Law in integral form, find the $\vec{E}$-field of the infinitely long straight wire.

2. Consider a *charged sphere* of radius R with a spherically-symmetric charge distribution

$$\rho(r) = kr^{\pi}, \tag{3}$$

for some constant k. Here we wish to ultimately find the $\vec{E}$-field *inside* the sphere itself, so we consider a Gaussian sphere of radius r where $r < R$, as shown in the figure below.



a) Calculate the total charge enclosed within the Gaussian surface, Q_{enc}.[1]

b) Using spherical-polar coordinates, construct the differential area element, $d\vec{a}$, for the Gaussian surface.

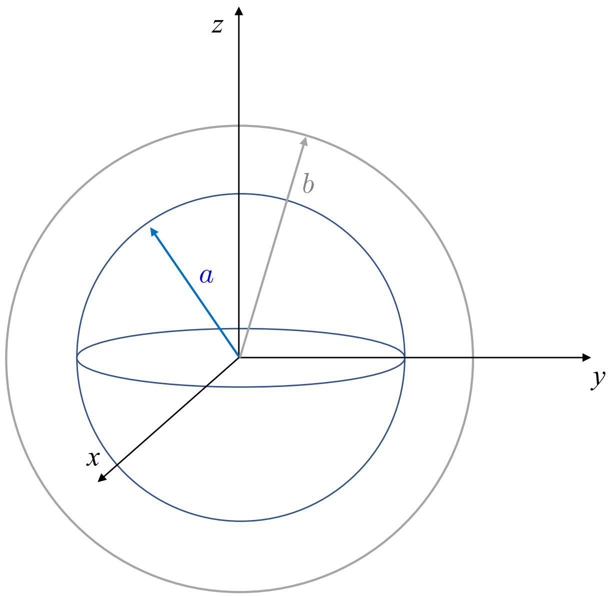
c) Using Gauss' Law in integral form, find the $\vec{E}$-field of the charged sphere for $r < R$.

[1] *Hint:* This charge density is *not* uniform; you *must* integrate to get the enclosed charge.

3. Consider a *hollow spherical shell* with a spherically-symmetric charge distribution

$$\rho(r) = kr^{\pi} \quad \text{for} \quad a \leq r \leq b \tag{4}$$

for some constant k.



Using Gauss' law in integral form, find the $\vec{E}$-field in each of the three regions:

a) $r < a$.

b) $a < r < b$.

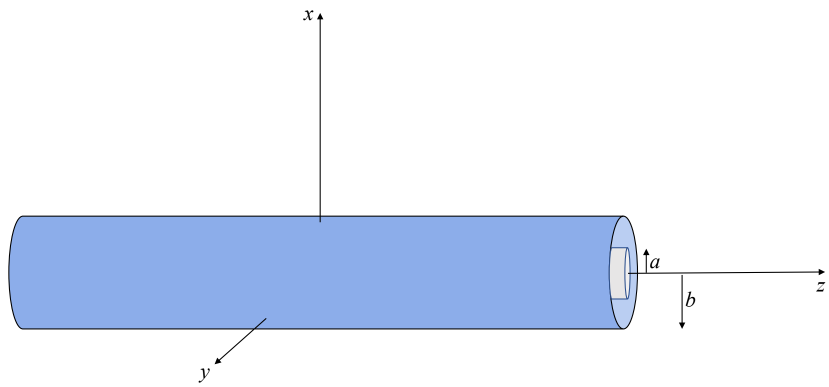
c) $r > b$.

4. A long coaxial cable carries a *volume* charge density

$$\rho(s) = \rho_0 \frac{s}{a} \quad \text{for} \quad s \leq a \tag{5}$$

in the inner cylinder and a *uniform surface* charge density on the outer cylindrical shell

$$\sigma(s) = \sigma_0 \quad \text{at} \quad s = b. \tag{6}$$



Notice that the surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral.

Using Gauss' law in integral form, find the $\vec{E}$-field in each of the three regions:

a) $s < a$.

b) $a < s < b$.

c) $s > b$

d) Find an expression for the surface charge density, σ_0, in terms of ρ_0, a, and b.

5. Consider a vector function of the form

$$\vec{E}(x, y, z) = (2xy + 3x^2 z)\ \hat{x} + x^2\ \hat{y} + x^3\ \hat{z}. \tag{7}$$

a) Show that Eq. (7) has zero curl and therefore can represent an electrostatic $\vec{E}$-field.

An electrostatic potential describing an $\vec{E}$-field can be found by calculating the line integral of the vector field

$$V(\vec{r}\,) = -\int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot d\vec{\ell}. \tag{8}$$

Calculate the line integral of Eq. (7) from the origin to point $\vec{r} = (x, y, z)$ by two different routes:

b) $(0,0,0)\ \rightarrow\ (x,0,0)\ \rightarrow\ (x,y,0)\ \rightarrow (x,y,z),,$

c) $(0,0,0)\ \rightarrow\ (0,0,z)\ \rightarrow\ (0,y,z)\ \rightarrow (x,y,z);$

d) Calculate the gradient of the electrostatic potential found in parts b) and c) and show that it yields Eq. (7).