Physics 311

Homework Set 5

Due Monday, September 19

1. Consider four point charges located at the corners of a square with charges and coordinates

$$q_1 = q \ @ \ x = y = 0 \quad , \quad q_2 = 2q \ @ \ x = \ell \ , \ y = 0$$
$$q_3 = -q \ @ \ x = y = \ell \quad , \quad q_4 = 3q \ @ \ x = 0 \ , \ y = \ell \tag{1}$$

a) Draw a diagram showing the position of each of the charges.

b) Draw a free-body diagram for charge q_1 due to the three other charges.

c) Calculate the force on q_1 due q_2, on q_1 due to q_3, and on q_1 due to q_4, in *unit-vector notation.*

d) Calculate the net force on q_1 due to the other charges in *unit-vector notation.*

e) Calculate the net force on q_1 due to the other charges in *magnitude-angle notation.*

Answer:

$$\vec{F}_1 = \left(\left[13 + \frac{1}{4} - \frac{5}{\sqrt{2}} \right]^{1/2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{\ell^2}, \ 238.1^\circ \right) \tag{2}$$

2. a) Calculate the net charge on a solid sphere of radius R with a volume charge density

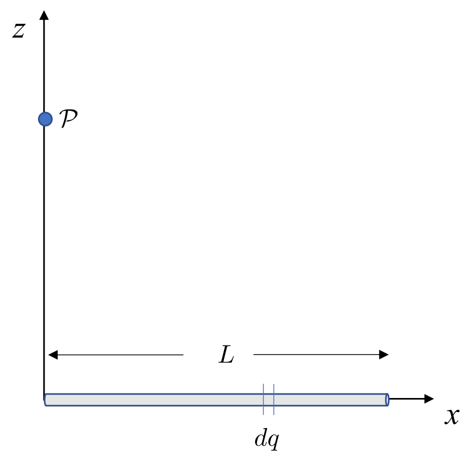
$$\rho(\theta) = \rho_0 \cos^2 \theta, \tag{3}$$

where θ is the polar angle.

b) Calculate the net charge on a circular plate of radius R with a surface charge density

$$\sigma(s) = \sigma_0 \frac{R}{s}. \tag{4}$$

3. Consider a line charge of length L lying on the x-axis with its left end at the origin, as shown in the figure below. In this problem, we ultimately wish to construct the total $\vec{E}$-field of this line charge at point $\mathcal{P}$ in terms of an integral.



a) On the diagram, *draw* the position vector $\vec{r}$ of the *field point*, $\mathcal{P}$, the position vector $\vec{r}\,'$ of the *source point*, dq, and the separation vector $\vec{\mathscr{r}}$ from the source point to the field point.

b) In unit-vector notation, construct the separation vector, $\vec{\mathscr{r}}$, and the unit separation vector, $\hat{\mathscr{r}}$.

c) Starting with the fundamental expression

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}, \tag{5}$$

the results from b), and the fact that $dq = \lambda(x)dx$, construct the total $\vec{E}$-field in terms of an integral.

4. Reconsider the previous problem for a line charge with a uniform density

$$\lambda(x) = \lambda_0. \tag{6}$$

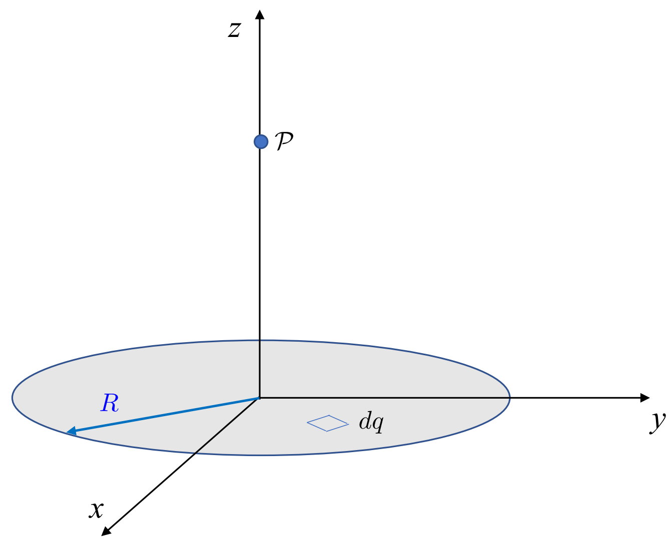
a) Calculate the total $\vec{E}$-field at point $\mathcal{P}$.

Answer:

$$\vec{E}(z) = \frac{\lambda_0}{4\pi\epsilon_0}\left[\frac{L}{z\sqrt{L^2+z^2}}\,\hat{z} - \left[\frac{1}{z} - \frac{1}{\sqrt{L^2+z^2}}\right]\hat{x}\right] \tag{7}$$

b) Show that in the limit that $z \gg L$, Eq. (7) reduces to that of a point charge.

5. Consider a charged, flat circular disk of radius R lying on the xy-plane with its center at the origin, as shown in the figure below. In this problem, we ultimately wish to construct the total $\vec{E}$-field on this charged disk at point $\mathcal{P}$ in terms of an integral.



a) On the diagram, *draw* the position vector $\vec{r}$ of the *field point*, $\mathcal{P}$, the position vector $\vec{r}\,'$ of the *source point*, dq, and the separation vector $\vec{\mathscr{r}}$ from the source point to the field point.

b) Using cylindrical coordinates, construct the separation vector, $\vec{\mathscr{r}}$, and the unit separation vector, $\hat{\mathscr{r}}$ in unit-vector notation.

c) Using the fact that $dq = \sigma(s)da$ for an area charge density that only depends on the distance from the z-axis, construct the total $\vec{E}$-field in terms of an integral. The differential area, da, should be written in terms of cylindrical coordinates.

6. Reconsider the previous problem for an areal charge with a uniform density

$$\sigma(s) = \sigma_0. \tag{8}$$

a) Calculate the net charge on the circular plate of radius R.

b) Calculate the total $\vec{E}$-field at point $\mathcal{P}$.

Answer:

$$\vec{E}(z) = \frac{\sigma_0}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]\,\hat{z} \tag{9}$$

c) Show that in the limit that $z \gg R$, Eq. (9) reduces to that of a point charge.

d) Show that in the limit that $z \ll R$, Eq. (9) reduces an infinite sheet of charge.

7. Suppose the electric field in some region is found to be

$$\vec{E} = kr^5\hat{r}, \tag{10}$$

in spherical-polar coordinates, where k is a constant.

a) Calculate the charge density $\rho(r)$.

b) Using the fact that

$$\rho = \frac{dq}{d\tau}, \tag{11}$$

find the total charge contained in a sphere of radius R, centered at the origin.

c) Using Gauss' Law in integral form

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}, \tag{12}$$

find the total charge contained in a sphere of radius R, centered at the origin.