Laboratory 3: Electric Potential Difference

Work and energy, which permeate classical physics, are useful for describing many electrostatic situations. Additionally in the same way that electrostatic forces can be determined via electric fields, electric potential energies can be determined via a quantity called electric potential.

Much of this exercise is based on a similar exercise in *Tutorials in Introductory Physics* by McDermott and Shaffer.

1 Work and Energy

Suppose that several forces act on an object which moves in a straight line through a known distance. Consider one of these forces, $\vec{F}$, which is known to be constant throughout the object’s motion, as illustrated on the left.

The work done by this force is

$$W = F \Delta x \cos \theta$$  \hspace{1cm} (1)

where $F$ is the magnitude of the force, $\Delta x$ is the length of the displacement and $\theta$ is the angle between the force and displacement vectors as illustrated on the right.

a) In each of the three scenarios illustrated below an object travels to the right along the indicated straight line. In each case describe whether the work done by the illustrated force is positive, negative or zero. *This will form a reference for the rest of this exercise.*
If the force or the direction of motion varies, then the motion can be broken down into infinitesimally small portions during which the force and direction of motion are approximately constant. An infinitesimal contribution to the work can be calculated for each portion and these can be added to give the work for the entire motion.

If multiple forces act on an object, then the work done by each force can be computed independently of the other forces. The net work, $W_{\text{net}}$, is the sum of all the individual works done by the individual forces. The work-kinetic energy theorem states that

$$\Delta K = W_{\text{net}}$$

where the kinetic energy of an object of mass $m$, moving with speed $v$ is

$$K = \frac{1}{2}mv^2.$$  

You will now check this qualitatively. The purpose of these exercises is to determine the work done and change in kinetic energy independently and then verify that these agree, according to Eq. (2). Thus in these exercises you cannot use Eq. (2) until you have determined the work done and the change in kinetic energy.

b) Suppose that exactly two forces act on a particle which moves from an initial to a final location as illustrated. The force vectors are drawn to scale.

Without using the notion of kinetic energy, describe whether the object’s speed increases, decreases or stays constant. Explain your answer.
Determine whether the work done by each of the two forces is positive or negative and whether the net work done is positive or negative.

Describe whether your answers to the last two questions are consistent with the work-kinetic energy theorem.

From this point on, you can use Eq. (2) to arrive at or explain your answers.

c) Suppose that exactly four forces act on a particle which moves from an initial to a final location as illustrated.

For each force determine whether the work done is positive, negative or zero. Is the net work positive, negative or zero? Does the object speed up or slow down? Explain your answers.
2 Work and Potential Energy in Electrostatics

You will now consider work and potential energy in electrostatics in the context of two point charges. One of these, the source charge, is fixed and is regarded as producing the forces under consideration. The other, the probe (or test) charge is mobile and experiences forces exerted by the source charge as well as possibly by some other objects.

a) Suppose that the source and probe charges are positive. The probe charge is initially at rest and is released. After this the only object with which it interacts is the source charge. The probe charge moves in a straight line from an initial to a final location.

\[ \text{Source} \quad \text{Probe (initial)} \quad \text{Probe (final)} \]

Draw a vector for the force exerted by the source charge on the probe charge. Use this to determine whether the net work done on the probe charge is positive or negative. Describe whether this is consistent with the work-kinetic energy theorem.

b) In a different scenario a negative probe charge follows the indicated circular trajectory around a positive source charge.

\[ \text{Probe (final)} \]

\[ \text{Source} \quad \text{Probe (initial)} \]

Determine whether the work done on the probe charge by the force exerted by the source charge is positive, negative or zero. Describe whether the probe charge’s
speed increases, decreases or stays constant.

c) The same negative probe charge can be made to follow one of each of the two illustrated trajectories around the positive source charge.

Describe whether the work done by the force exerted by the source charge for the dashed trajectory is the same as, smaller than or larger than that for the solid trajectory. Explain your answer.
This is an example of the fact that the work done by an electrostatic force only depends on the initial and final locations of the object. In all such cases it is possible to define a potential energy. Here, this is called the electric potential energy, $U_{\text{elec}}$, and is defined so that

$$\Delta U_{\text{elec}} = -W_{\text{elec}}$$

where $W_{\text{elec}}$ is the work done by the electrostatic force.

d) Consider a positive probe charge which moves in the illustrated trajectory in the presence of a positive source charge.

Determine whether the work done and the change in electric potential energy by the source charge is positive, negative or zero for each leg. Enter these in the table.

<table>
<thead>
<tr>
<th>Segment</th>
<th>$W_{\text{elec}}$</th>
<th>$\Delta U_{\text{elec}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A → B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B → C</td>
<td></td>
<td></td>
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<tr>
<td>C → D</td>
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<tr>
<td>D → A</td>
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<tr>
<td>Entire loop</td>
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</tbody>
</table>
e) A positive probe charge moves away from a positive source charge. Describe whether the electric potential energy increases, decreases or stays the same during this process?

f) A negative probe charge moves away from a positive source charge. Describe whether the electric potential energy increases, decreases or stays the same during this process?

3 Electric Potential

In electrostatics, the force exerted on a probe particle can be determined by using the electric field produced by the source charges, via

\[ \vec{F} = q\vec{E} \]  \hspace{1cm} (4)

where \( q \) is the charge of the probe particle. A similar representation can be attained when considering energy in electrostatics.

a) An arrangement of source charges produces a uniform electric field and a probe particle, with charge \( q \) moves from an initial to final location as illustrated.
A certain amount of work is done by the electric field on the charge in this scenario. Suppose that the charge is replaced by another with exactly double the charge and it moves between the same points. How is the work in this new scenario related to that in the first scenario? What if the charge were tripled or multiplied by 5 times? Answer these as precisely as possible (increases, decreases or stays the same is not enough). Explain your answer.

b) Repeat the above scenario for the case where the probe charge is replaced by one whose charge has the same magnitude but opposite sign.

c) The previous questions should have convinced you that the work done on the probe charge is proportional to its charge. What does this imply for the way in which the electric potential energy depends on the charge of the probe? Explain your answer.

Thus, for a probe charged particle, whose charge is $q$, changes in the electric potential energy can be expressed as

$$\Delta U_{\text{elec}} = q\Delta V$$  \hspace{1cm} (5)$$
where $V$ is the electric potential established by the source charges and this is independent of the probe charge.
d) Consider a positive probe charge which moves in the illustrated trajectory in the presence of a positive source charge.

\[ \text{Source} \quad \rightarrow \quad \text{Probe} \]

Determine whether the change in electric potential is positive, negative or zero for each leg. Enter these in the table.

<table>
<thead>
<tr>
<th>Segment</th>
<th>$\Delta V$</th>
</tr>
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<td>A $\rightarrow$ B</td>
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<tr>
<td>B $\rightarrow$ C</td>
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<td>Entire loop</td>
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</table>

e) Would your answers to the previous question be any different if the sign of the probe charge were opposite to what you used in the previous part? Explain your answer. BE CAREFUL!
f) Considering the previous diagram, suppose that a positively charged particle is released from rest at point B and moves to point C. The mass of the particle is $2.0 \times 10^{-3}$ kg, its charge is $4.0 \times 10^{-6}$ C and its speed at point C is 8.0 m/s. Determine the change in kinetic energy of the particle and the change in electric potential between B and C.

g) Another particle of the same mass but with four times the charge is released at B. What will its speed at C be?