1 Motion diagrams: horizontal motion

A car moves to the right. For an initial period it slows down and after that it speeds up. Which of the following (choose one) best represents its location as time passes?

<table>
<thead>
<tr>
<th>Case 1</th>
<th>1 s</th>
<th>2 s</th>
<th>3 s</th>
<th>4 s</th>
<th>5 s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2</th>
<th>1 s</th>
<th>2 s</th>
<th>3 s</th>
<th>4 s</th>
<th>5 s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3</th>
<th>1 s</th>
<th>2 s</th>
<th>3 s</th>
<th>4 s</th>
<th>5 s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 4</th>
<th>1 s</th>
<th>2 s</th>
<th>3 s</th>
<th>4 s</th>
<th>5 s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

Briefly explain your choice.
2 Motion diagrams and position vs. time graphs

A car moves from left to right and its position, measured in meters, is recorded every 5.0 s. The resulting motion diagram is illustrated.

a) Produce a table of numerical data for position versus time for the car for the duration of the motion.
b) Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

3 Motion diagrams and position vs. time graphs

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b) Produce a position versus time graph for the car for the duration of the motion. This graph must be drawn by hand using axes that are clearly labeled.

4 Average velocity

The following objects lie along a straight line: a bicycle, a coffee cup and a soccer ball. The distance from the coffee cup to the bicycle is 400 m and from the cup to the ball is 500 m. A man starts at the cup and travels in a straight line to the ball. This takes 200 s. A dog is initially at the cup and runs at constant speed to the bicycle, taking 50 s to do so. The dog immediately turns around and runs to ball; this takes the dog an additional 150 s. Consider the entire trip from the cup to the ball for each. Who has the larger average velocity for this entire trip? Explain your answer.
5 **Ant on a stick**

An ant walks along a straight stick. The graph illustrates the ant’s position vs. time. Answer the following, giving explanations for each answer.

a) During which times is the ant moving right? During which times is it moving left?

b) When, if ever, is the velocity of the ant 0 m/s?

c) How does the speed of the ant at 1.0 s compare to its speed at 4.0 s?

d) How does the velocity of the ant at 1.0 s compare to its velocity at 4.0 s?

6 **Ant and bug on a stick**

An ant and a bug walk along straight sticks. The solid graph illustrates the ant’s position vs. time. The dashed graph indicates the bug’s position vs. time. Answer the following, giving explanations for each answer.

a) At what time(s) are the ant and bug at the same location?

b) Which is moving faster at 2 s?

c) Do the ant and bug ever have the same velocity? If so when?
7 Acceleration sign

A bicycle can move east (positive) or west (negative).

a) If the bicycle moves east can the acceleration be negative? Explain your answer.
b) If the bicycle moves west can the acceleration be positive? Explain your answer.

8 Ant and bug on a stick

An ant and a bug walk along straight sticks. The solid graph illustrates the ant’s position vs. time. The dashed graph indicates the bug’s position vs. time. For the bug, and separately for the ant, which of the following is true during the period from 0s to 4s?

i) Acceleration is zero at all times.
ii) Acceleration is positive at all times.
iii) Acceleration is negative at all times.
iv) Acceleration is first positive and later negative.
v) Acceleration is first negative and later positive.
9 Non-Freely Falling Object

A bungee jumper falls downward stretching the cord, reaching a low point, after which the cord pulls him up again. His velocity is recorded at equally spaced intervals in time. The data is:

<table>
<thead>
<tr>
<th>Time in s</th>
<th>Velocity in m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>−20.0</td>
</tr>
<tr>
<td>10.5</td>
<td>−15.0</td>
</tr>
<tr>
<td>11.0</td>
<td>−10.0</td>
</tr>
<tr>
<td>11.5</td>
<td>−5.0</td>
</tr>
<tr>
<td>12.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12.5</td>
<td>5.0</td>
</tr>
<tr>
<td>13.0</td>
<td>10.0</td>
</tr>
<tr>
<td>13.5</td>
<td>15.0</td>
</tr>
<tr>
<td>14.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

a) During which period is the man falling? When is he rising?
b) By how much does the man’s velocity change per second? Is this change constant throughout the recorded motion?
c) Determine the man’s acceleration while he is falling and also while he is rising. Are these accelerations the same or not?
d) What is the man’s acceleration (according to the data) at his low point?
Go to the moving man animation at:
http://phet.colorado.edu/en/simulation/moving-man

Run the moving man animation. Click on the charts tab. Set the position to 0.00 m, the velocity to $-5.00 \, \text{m/s}$ and the acceleration to $2.00 \, \text{m/s}^2$. Run the animation, stopping it just before the man hits the wall. The animation will have recorded the motion. Check the playback button at the bottom. You can slide the light blue bar left and right to get data for the motion. Gray zoom icons at the right will let you rescale the charts.

a) Consider the interval from 2.0 s to 3.0 s. Describe the motion verbally during this time.
b) How does the speed of the man at 2.0 s compare to that at 3.0 s during this period? Explain your answer.
c) How does the velocity of the man at 2.0 s compare to that at 3.0 s during this period? Explain your answer.
d) Will the average acceleration over the interval from 2.0 s to 3.0 s be positive, negative or zero? Explain your answer.
e) If the acceleration is not zero, does it vary during this interval? Explain your answer.
f) Determine the average acceleration over the interval from 2.0 s to 3.0 s.
11 Acceleration vector

a) A hockey puck slides along a horizontal surface toward a board, hitting it at an angle and bouncing off with unchanged speed. The view from above is as illustrated. Draw the velocity vectors of the puck just before and just after hitting the board, use these to draw the vector \( \Delta \vec{v} \), and use the result to draw the direction of the acceleration vector.

b) If the puck traveled backwards along the same path (i.e. reversed direction), what would the direction of the acceleration vector be?

12 Ants moving along a curved path

Various ants follow the same path on a horizontal surface, starting at point 1. The path is as illustrated. Ant A moves with a constant speed, ant B gradually speeds up and ant C gradually slows.

a) Draw the velocity vector at points 1, 2, 3 and 4.

b) Does any of the ants have zero acceleration at all times? Explain your answer.

13 Hockey pucks sliding horizontally

Three identical hockey pucks slide horizontally across a frictionless sheet of ice and they maintain the indicated speeds while they are being observed. Let \( F_A \) be the magnitude of the force acting on A, \( F_B \) be the magnitude of the force acting on B, etc, .... Which of the following is true? Explain your answer.

i) \( F_B = 2F_A \) and \( F_C = 4F_A \)

ii) \( F_A = 2F_B \) and \( F_A = 4F_C \)

iii) \( F_A = F_B = F_C \neq 0 \)

iv) \( F_A = F_B = F_C = 0 \)
14 Moving carts

Three identical carts move horizontally along tracks. Their speeds at two instants 5.0 s apart are indicated. Let $F_A$ be the magnitude of the force acting on A during this interval, $F_B$ be the magnitude of the force acting on B, etc, ..., Which of the following is true? Explain your answer.

i) $F_A > F_B > F_C$.

ii) $F_B = F_C > F_A$.

iii) $F_B = F_C < F_A$.

iv) $F_A = F_B = F_C \neq 0$

---

15 Pushing carts

Zog and Geraldine (his wife) each push a cart along a horizontal surface where friction is negligible. Both carts are initially at rest. Zog takes the cart with mass 25 kg and exerts a force of 400 N on it for a period of 4.0 s and he then collapses and stops pushing. Geraldine has to push a cart of mass 50 kg and she is also able to exert a force of 400 N on it. Geraldine claims that it is possible for the speed of her cart to eventually reach the speed of Zog’s cart. Is this true? Explain your answer.
A Suspended Object in Equilibrium

A 2.50 kg ring is suspended from the ceiling and is held at rest by two ropes as illustrated. Rope 2 pulls horizontally. The aim of this exercise is to use Newton’s 2nd Law to determine the tension in each rope. One piece of background information that you will need to answer this is that the magnitude of the gravitational force on an object of mass $m$ is $F_g = mg$.

a) Draw a free body diagram for the ring. Label the tension forces $\overrightarrow{T_1}$ and $\overrightarrow{T_2}$.

b) Write Newton’s 2nd Law in its component form, i.e. write

$$F_{net \, x} = \Sigma F_{ix} = ma_x \tag{1}$$
$$F_{net \, y} = \Sigma F_{iy} = ma_y \tag{2}$$

Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the acceleration and the normal force.

c) List as much information as possible about each component for each force, using one of the two formats below.

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$ comp</th>
<th>$y$ comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{gx}$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$F_{gy}$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$T_{1x}$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$T_{1y}$</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$ comp</th>
<th>$y$ comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overrightarrow{F_g}$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$\overrightarrow{T_1}$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
</tbody>
</table>

d) Express the components of $\overrightarrow{T_2}$ in terms of its magnitude, $T_2$. Repeat this for $\overrightarrow{T_1}$ and insert the expressions into the table or equations above.

e) Use Eq. (1) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (2). You should get two expressions that contain the two unknowns $T_1$ and $T_2$. Solve them for the unknowns.

f) If you had one rope that is rated to break when the tension exceeds 30 N and another rated to break when the tension exceeds 40 N, which one would you use to suspend the object as illustrated above?
Free fall in an elevator

A phone of mass \( m \) sits on the floor of an elevator, which is initially at rest. The elevator cable snaps and the elevator and phone then undergo free fall. While they do this which is true of the magnitude of the normal force, \( n \), acting on the phone? Explain your choice.

i) \( n = 0 \).
ii) \( mg > n > 0 \).
iii) \( n = mg \).
iv) \( n > mg \)
a) Draw a free body diagram for the block.
b) Write Newton’s Second Law in its component form, i.e. write

\[ F_{\text{net, x}} = \sum F_x = \cdots \]  
\[ F_{\text{net, y}} = \sum F_y = \cdots \] (3) (4)

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations shortly; they will generate the algebra that eventually gives you the acceleration and the normal force.
c) Determine the magnitude of the gravitational force. Let \( n \) be the magnitude of the normal force. Do you think that \( n = mg \)?
d) List all the components of all the forces, using one of the two formats below.

\[
\begin{array}{|c|c|c|}
\hline
\text{Force} & x \text{ comp} & y \text{ comp} \\
\hline
\vec{F}_g & \cdots & \cdots \\
n & \cdots & \cdots \\
\hat{n} & \cdots & \cdots \\
\vdots & \cdots & \cdots \\
\hline
\end{array}
\]
e) Use Eq. (3) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (4). Solve these for the acceleration and the magnitude of the normal force. Is \( n = mg \)?
f) Suppose that rather than pull up, the person pushed down on the box at the same angle from the left and with the same force. Would the acceleration and normal forces differ from the case where the person pulled up?
g) You may have noticed that the acceleration does not depend on the normal force. This is only true if there is no friction. It turns out that when friction is present, the magnitude of friction force increases as the normal force increases. Knowing this, would pulling up or pushing down give a larger acceleration?
19 Dynamics of a Single Object with Friction

A box of mass 15.0 kg can move along a horizontal surface. A person pulls with a force at the illustrated angle. The coefficient of kinetic friction is 0.25. The primary aim of this exercise will be to determine the acceleration of the box.

a) Draw a free body diagram for the block.
b) Write Newton’s Second Law in its component form, i.e. write

\[ F_{\text{net}}^x = \Sigma F_x = \cdots \]  \hspace{1cm} (5)
\[ F_{\text{net}}^y = \Sigma F_y = \cdots \]  \hspace{1cm} (6)

Insert as much information as possible about the components of acceleration at this stage. You will return to these equations shortly; they will generate the algebra that eventually gives you the acceleration and the normal force.

c) Determine the magnitude of the gravitational force. Let \( n \) be the magnitude of the normal force. Using this write an expression for the magnitude of the friction force. Do you know the exact number for the friction force at this point?

d) List all the components of all the forces, using one of the two formats below.

\[
\begin{align*}
F_{gx} &= \cdots \\
F_{gy} &= \cdots \\
n_x &= \cdots \\
n_y &= \cdots \\
\vdots
\end{align*}
\]

\[
\begin{array}{ccc}
| \text{Force} | x \text{ comp} | y \text{ comp} | \\
| \hline \\
| \vec{F}_g | & & \\
| \vec{n} | & & \\
| \vdots | & & \\
\end{array}
\]

e) Use Eq. (5) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (6). Does either give the acceleration immediately? Can one of them at least give the normal force immediately?

f) Determine the normal force and use this result to find the acceleration.

g) Suppose that the box were initially at rest. Determine the time taken to pull it a distance of 5.0 m.
20 Dynamics of an Object on a Ramp

A 4.0 kg box can move along a frictionless ramp which makes at angle 30° from the horizontal. A person pulls on a rope which exerts a force of 15 N up the ramp parallel to its surface. The object of this exercise is to determine the acceleration of the box.

a) Draw a free body diagram for the box.
b) Describe the x and y axes that you will use.
c) Write Newton’s Second Law in vector form and also in its component form, i.e. write

\[ F_{\text{net}} x = \Sigma F_x = \cdots \]  \hspace{1cm} (7)
\[ F_{\text{net}} y = \Sigma F_y = \cdots \]  \hspace{1cm} (8)

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.
d) Determine the magnitude of the weight force.
e) List all the components of all the forces, using one of the two formats below.

\[ W_x = \cdots \]
\[ W_y = \cdots \]
\[ n_x = \cdots \]
\[ n_y = \cdots \]
\[ \vdots \]

f) Use Eq. (7) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (8). Use the resulting equations to determine the acceleration of the box.
g) Is it possible to say with certainty whether the box is moving up the ramp or down the ramp? Is either direction possible in this situation? If only one direction is possible, which is it?
21 Dynamics of an Object on a Ramp with Friction

A box can move along a rough ramp which makes at angle $\theta$ from the horizontal and which has length $L$. The box is launched with speed $v$ from the top of the ramp. The aim of this exercise is to determine the coefficient of friction needed to bring the box to a stop at the bottom of the ramp.

a) The first part of the solution uses kinematics to assess the acceleration of the box. Using kinematics, and eventually dynamics, is greatly simplified by choosing an appropriate “$x$” and a “$y$” axis. These do not have to be along the usual vertical and horizontal directions. Regardless of the axes that you choose, the usual general kinematics and dynamics equations will be valid. Describe the “$x$” and “$y$” axes that you will use.

b) Determine an expression for the magnitude of the acceleration of the box, $a$, in terms of variables relevant to this problem, such as $L$, $\theta$, $v$, and possibly the mass of the box, $m$.

c) Draw a free body diagram for the box.

d) Write Newton’s Second Law in vector form and also in its component form, i.e. write

$$F_{net \, x} = \Sigma F_{ix} = \cdots$$

$$F_{net \, y} = \Sigma F_{iy} = \cdots$$

(9)

(10)

where $x$ and $y$ refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

e) Determine expressions for the magnitudes of the gravitational and the friction forces.

f) List all the components of all the forces, using one of the two formats below.

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$ comp</th>
<th>$y$ comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{F}_g$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{n}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


g) Use Eq. (9) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (10). Use the resulting equations to find an expression for the coefficient of kinetic friction. Does the result depend on the mass of the box?

h) Suppose that the ramp is a roof whose length is 5.0 m and which is angled at $15^\circ$ from the horizontal. If the box is pushed with speed 4.0 m/s, determine the coefficient of friction needed to stop the box at the bottom of the roof.
22 Dynamics of an Object on a Ramp

A block of mass $m$ slides up a ramp which makes at angle $\theta$ from the horizontal. The coefficient of kinetic friction between the surfaces is $\mu_k$. A person pushes horizontally with a force $F_p$. The object of this exercise is to determine an expression for the acceleration of the block

\[ a = \text{formula involving } m, \theta, F_p, \mu_k, g \text{ and constants.} \]

The entire collection of these steps is called “applying Newton’s laws of mechanics to determine the acceleration of the block.”

a) Draw a free body diagram for the block.

b) Describe the $x$ and $y$ axes that you will use.

c) Write Newton’s Second Law in vector form and also in its component form, i.e. write

\[
\begin{align*}
F_{\text{net}} x &= \cdots \quad (11) \\
F_{\text{net}} y &= \cdots \quad (12)
\end{align*}
\]

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

d) Determine expressions for the magnitudes of all the friction and gravitational forces. Do you think that $n = mg$?

e) List all the components of all the forces, using one of the two formats below.

\[
\begin{array}{c|c|c}
\hline
\text{Force} & x \text{ comp} & y \text{ comp} \\
\hline
F_{gx} & \cdots & \cdots \\
F_{gy} & \cdots & \cdots \\
 & \cdots & \cdots \\
\cdots & \cdots & \cdots \\
\hline
\end{array}
\]

f) Note that $F_{\text{net}} x = \sum_i F_{ix}$ (i.e. the $x$ component of the net force is the sum of the $x$ components of individual forces). Use this and Eq. (11) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (12). Use the resulting algebraic expressions to get an expression for $n$. Is this $mg$? Use the resulting equations to get an expression for $a$. 

15
23 Connected Objects: Tension and Acceleration

Two boxes can move along a horizontal surface. There is no friction between either box and the surface. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N.

a) Determine the acceleration of each box.
b) Determine the tension in the rope connecting the boxes.

24 Connected Objects: Friction

Two boxes can move along a horizontal surface. There is no friction between the 6.0 kg box and the surface. There is friction for the other box: the coefficient of static friction is 0.70 and the coefficient of kinetic friction is 0.50. The boxes are connected by a rope. A hand pulls on the other rope with force 50 N.

a) Determine the acceleration of each box.
b) Determine the tension in the rope connecting the boxes.

25 Dynamics of Connected Objects; Level/Suspended Blocks without Friction

Two blocks are connected by a string, which runs over a massless pulley. A 10 kg block is suspended and a 5.0 kg block can slide along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. The blocks held at rest and then released. They move, constantly speeding up. Which of the following is true regarding the tension in the connecting string, $T$, while they move? Explain your choice.

i) $T = 0$.
ii) $98 \text{ N} > T > 0$.
iii) $T = 98 \text{ N}$.
iv) $T > 98 \text{ N}$.
Two blocks are connected by a string, which runs over a massless pulley. One block, of mass $m_1$, is suspended and the other block, of mass $m_2$, can move along a frictionless horizontal surface. The string connected to the block on the surface runs horizontally. A hand exerts a constant force on the block on the surface. This force has magnitude $F_{\text{hand}}$ and points horizontally to the left. Determine an expression for the magnitude of acceleration of the blocks, $a$. This should be of the form

$$a = \text{expression with only } m_1, m_2, g, F_{\text{hand}} \text{ and constants}$$

a) Consider the block on the horizontal and carry out the following.
   i) Draw a free body diagram for this block.
   ii) Write Newton’s second law in vector form for this block. Rewrite this in vertical and horizontal component form. Note: Just writing $F = ma$ is not completely correct and is too imprecise to eventually give a correct answer.
   iii) Use the free body diagram to rewrite the component form of Newton’s second law in terms of the individual forces acting on this block. Can you manipulate this to obtain

$$a = \text{formula involving only } m_1, m_2, g \text{ and constants?}$$

b) Repeat part a) for the suspended block.

c) Combine the expressions obtained for each block to obtain a single expression for the acceleration of the block on the horizontal.
27 Dynamics of Connected Objects: Level/Suspended Blocks with Friction

Two blocks are connected by a string, which runs over a massless pulley. One block can move along a rough horizontal surface; the other is suspended. The string connected to the block on the surface runs horizontally. Ignore air resistance on either block. The aim of this exercise will be to determine the acceleration of the blocks.

Suppose that mass of the suspended block is 6.0 kg and the mass of the block on the surface is 4.0 kg. The coefficient of friction between the block and surface is 0.25. First consider the block on the surface

a) Draw a free body diagram for the block on the surface.

b) Write Newton’s Second Law in component form for the block on the surface, i.e. write

\[ F_{\text{net} x} = \Sigma F_{\text{ix}} = \cdots \]  
\[ F_{\text{net} y} = \Sigma F_{\text{iy}} = \cdots \]  

(13)  
(14)

Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

c) List all the components of all the forces for the block on the surface.

\[ F_{gx} = \cdots \]  
\[ F_{gy} = \cdots \]  
\[ n_x = \cdots \]  
\[ n_y = \cdots \]  
\[ \vdots \]

d) Use Eqs. (13) and (14) and the components to obtain an expression for the acceleration of the blocks. Can you solve this for acceleration at this stage?

e) Repeat parts a) to d) for the suspended block. Be careful about the acceleration!

f) Combine the equation for the two blocks to obtain the acceleration and the tension in the rope.

The analysis can be performed for blocks of any mass. Let \( m_1 \) be the mass of the block on the surface, \( m_2 \) the mass of the suspended block and \( \mu_k \) be the coefficient of friction between the block and the surface.

g) Determine an expression for the magnitude of acceleration of the blocks, \( a \). This should be of the form

\[ a = \text{formula with only } m_1, m_2, g, \mu_k \text{ and constants.} \]
28 Dynamics of Connected Objects: Atwood’s Machine

Two blocks, with masses indicated, are connected by a string which runs over a massless pulley. Use the following steps to determine an expression for the magnitude of acceleration of the blocks, \( a \). This should be of the form

\[ a = \text{formula involving only } m_1, m_2, g \text{ and constants.} \]

The entire collection of these steps is called “applying Newton’s laws of mechanics to determine the acceleration of the blocks.”

a) Consider the block on the left and carry out the following.
   i) Draw a free body diagram for the block on the left.
   ii) Write Newton’s second law in vector form for the block on the left. Rewrite this in vertical and horizontal component form. \( \text{Note: Just writing } F = ma \text{ is not completely correct and is too imprecise to eventually give a correct answer.} \)
   iii) Use the free body diagram to rewrite the component form of Newton’s second law in terms of the individual forces acting on the block on the left. Can you manipulate this to obtain

\[ a = \text{formula involving only } m_1, m_2, g \text{ and constants?} \]

b) Repeat part a) for the block on the right. Is the acceleration of the two blocks exactly the same? If not, how are the accelerations related? Convert this into a simple algebraic relationship. \( \text{Note: You should be convinced that it cannot be correct to use the same symbol to represent the vertical component of acceleration for each block.} \)

c) Combine the expressions obtained for each block to obtain a single expression for the vertical component of the acceleration of the block on the left. Use this to obtain an expression for the magnitude of acceleration of each block.
29 Dynamics of Connected Objects: Atwood’s Machine Variation

Two blocks, with masses indicated, are connected by a string which runs over a massless pulley. A hand exerts a constant downward force with magnitude $F_{\text{hand}}$ and the block on the left. Determine an expression for the magnitude of acceleration of the blocks, $a$. This should be of the form

$$a = \text{formula involving only } m_1, m_2, g, F_{\text{hand}} \text{ and constants.}$$
30 Bug walking in a circle

A bug walks at a constant speed in a circular path on a horizontal surface. Which vector best illustrates the net force on the bug at the illustrated moment? Explain your choice.

31 Ball Swinging in a Vertical Circle

A 0.20 kg ball swings with in a vertical circle at the end of a string of length 0.50 m.

a) Draw a free body diagram for the ball at the highest point of the circle. Draw a free body diagram at the lowest point.

b) In general the speed of the ball can vary as it swings. As the speed decreases does the tension at the top of the circle increase, decrease or stay constant? Determine the minimum speed so that the tension is not zero. Describe what happens if the speed drops beneath this.

c) Now suppose that the speed of the ball is constant throughout its motion. How does net force at the highest point of the circle compare (larger, smaller, same) to that at the lowest point of the circle? Use your answer to compare (larger, smaller, same) the tension in the string at the lowest point of the circle to the tension at the highest point of the circle.

d) Suppose that the string will break if the tension in it exceeds 5.0 N. Use Newton’s second law to analyze the situation where the tension is largest (i.e. highest or lowest point) and determine the maximum speed with which the ball can move so that the string does not break.
32 Block inside a Revolving Drum

A spinning drum has vertical wooden sides. A wooden block is placed inside the drum and the drum is eventually made to spin with a constant angular velocity. This is done in such a way that the block does not slide relative to the drum; it rotates at the same rate as the drum. In the first part of the problem suppose that the mass of the block is 2.0 kg, the coefficient of static friction is 0.60 and the radius of the drum is 0.80 m.

a) Draw a free body diagram for the block, identify the direction of the acceleration and write Newton’s 2nd law in component form. Is $n = mg$ in this case?

b) Determine an expression for the minimum angular velocity with which the drum must rotate so that the block does not slip.

Now consider the more general case where the drum has radius $r$ and the coefficient of static friction is $\mu_s$.

c) Determine an expression for the minimum angular velocity (in terms of $g$, $r$ and $\mu_s$) so that the block does not slip. Does it depend on the mass of the block?

Now suppose that the spinning drum has tilted sides. A block is placed inside the drum and the drum is eventually made to spin with a constant angular velocity $\omega$. This is done in such a way that the block does not slide relative to the drum; it rotates at the same rate as the drum. Let $r$ be the distance from the block to the axle of the drum.

d) Determine the minimum angular velocity required for the block not to slip when $\theta = 75^\circ$, $r = 2.5$ m and the coefficient of static friction is 0.2.
33 Half Pipe
A person of mass 60 kg is on a skateboard of mass 5.0 kg. Both are at rest at the top of a half-pipe of radius 10 m. Ignore any friction and the rotation of the wheels.

a) Determine the speed of the skateboarder at the bottom of the half pipe.
b) Suppose that there was another skateboarder, of mass 80 kg on a 5.0 kg skateboard at rest at the bottom of the pipe. The two skateboarders collide, hold each other and move together. Determine their speed moments after they collide. Use this to determine how high up the pipe they move.

34 Sledding
King Zog, with mass 160 kg, and Queen Geraldine, with mass 80 kg, sled down an icy hill. They start from rest at the same point above the bottom of the hill. Ignore friction and air resistance. Which of the following is true regarding their speeds at the bottom of the hill? Explain your answer.

i) Same speeds.
ii) Geraldine’s speed is twice that of Zog.
iii) Geraldine’s speed is four times that of Zog.
iv) Zog’s speed is larger than Geraldine’s speed.

35 Spring bumper
Two walruses (named X and Y), with the same masses, slide along horizontal sheets of ice. Each collides with a horizontal spring mounted to a wall; the springs are identical. Prior to hitting the spring, walrus X moved with speed twice that of walrus Y. The springs compress, bringing each walrus to a stop. Which of the following is true regarding the distances by which the springs compress? Explain your answer.

i) Springs compress by the same distance.
ii) X compresses spring by twice as much Y.
iii) X compresses spring by four times as much Y.
iv) X compresses spring by half as much Y.
v) X compresses spring by a quarter of what Y compresses.
36 Vectors: Dot Products

For each of the following, determine $A_x, A_y, B_x, B_y$ and $A_z$ and $B_z$ (if applicable) and determine the dot product, $\vec{A} \cdot \vec{B}$. Note: If your answer for the dot product contains $\hat{i}$ and $\hat{j}$ then it is very incorrect!

a)

\[\vec{A} = 2\hat{i} + 2\hat{j}\]
\[\vec{B} = 2\hat{i} + 2\hat{j}\]

b)

\[\vec{A} = 2\hat{i} + 2\hat{j}\]
\[\vec{B} = 2\hat{i} - 2\hat{j}\]

c)

\[\vec{A} = 2\hat{i} - 2\hat{j}\]
\[\vec{B} = 2\hat{i} + 2\hat{j}\]

d)

\[\vec{A} = 2\hat{i} - 2\hat{j}\]
\[\vec{B} = 2\hat{i} - 2\hat{j}\]

e)

\[\vec{A} = 2\hat{i} + 2\hat{j} + 3\hat{k}\]
\[\vec{B} = 2\hat{i} + 2\hat{j}\]

f)

\[\vec{A} = 2\hat{i} + 2\hat{j} + 3\hat{k}\]
\[\vec{B} = 2\hat{i} - 2\hat{j}\]

g)

\[\vec{A} = 2\hat{i} - 2\hat{j} + 3\hat{k}\]
\[\vec{B} = 2\hat{i} + 2\hat{j} + 3\hat{k}\]

h)

\[\vec{A} = 1\hat{i} - 2\hat{j} + 3\hat{k}\]
\[\vec{B} = 2\hat{i} + 1\hat{j} - 3\hat{k}\]
Answers:

a) $\vec{A} \cdot \vec{B} = 8$.
b) $\vec{A} \cdot \vec{B} = 0$.
c) $\vec{A} \cdot \vec{B} = 0$.
d) $\vec{A} \cdot \vec{B} = 8$.
e) $\vec{A} \cdot \vec{B} = 8$.
f) $\vec{A} \cdot \vec{B} = 0$.
g) $\vec{A} \cdot \vec{B} = 9$.
h) $\vec{A} \cdot \vec{B} = -9$. 
37 Vectors: Dot Products

For each of the following, express $\vec{A}$ in component form $\vec{B}$ using unit vectors and determine the dot product, $\vec{A} \cdot \vec{B}$. Note: If your answer for the dot product contains $\hat{i}$ and $\hat{j}$ then it is very incorrect!

a) 

Answer:

a) $\vec{A} \cdot \vec{B} = 6$.

b) 

c) $\vec{A} \cdot \vec{B} = -7$.

d) $\vec{A} \cdot \vec{B} = 6$. 
A monkey hangs from a spring which is attached to the ceiling of a building. The spring hangs vertically and the monkey bounces up and down without touching the floor.

a) As the monkey ascends toward and nears its highest point, the spring is compressed. Which of the following is true while this happens?

i) The spring does positive work, gravity does positive work.

ii) The spring does positive work, gravity does negative work.

iii) The spring does negative work, gravity does positive work.

iv) The spring does negative work, gravity does negative work.

b) As the monkey begins to descend away its highest point, the spring is still compressed. Which of the following is true while this happens?

i) The spring does positive work, gravity does positive work.

ii) The spring does positive work, gravity does negative work.

iii) The spring does negative work, gravity does positive work.

iv) The spring does negative work, gravity does negative work.
39 **Motion under a complicated potential**

A particle that can move along the $x$ axis is subjected to the potential

$$U(x) = \frac{(x^3 - 7.75x)}{5}.$$ 

a) Determine an expression for the horizontal component of the force on the object.

b) Determine locations where the object is in equilibrium and for each describe whether the equilibrium is stable or not.

40 **Hoisting fish**

King Zog and Queen Geraldine are fishing from a bridge and they catch identical twin fish, each with mass 5.0 kg. They hoist the fish at constant speeds to the bridge 8.0 m above the water. Zog takes 10 s to hoist his fish and Geraldine 7.5 s to hoist her fish.

a) Which of the following is true? Explain your answer.

   i) Zog and Geraldine do the same work.
   ii) Zog does more work than Geraldine.
   iii) Zog does less work than Geraldine.

b) Which of the following is true? Explain your answer.

   i) Zog and Geraldine expend the same power.
   ii) Zog expends less power.
   iii) Zog expends more power.

41 **Power delivered by engines**

Two engines pull identical objects along horizontal surfaces. Engine A delivers 2000 W of power and engine B delivers 4000 W. Engine A pulls for 5 min and engine B for 4 min. Which of the following is true.

   i) Engine A delivers more work than engine B.
   ii) Engine A delivers less work than engine B.
   iii) Engine A delivers the same work as engine B.
   iv) There is not enough information to decide.
42 Beam in equilibrium

A beam with mass $M$ and length $L$ is anchored to a wall and held at rest horizontally by a rope as illustrated. A ball with mass $m$ is suspended from the beam at the illustrated point. The aim of this exercise is to determine the tension in the rope. This would enable one to decide on the breaking strength of the rope.

a) State the conditions for equilibrium.

b) Draw all the force vectors on the beam.

c) Identify a pivot point (there are many correct possibilities – one is much more useful than the others) and determine expressions for the torque exerted by each force about the pivot.

d) Substitute the individual torques into one of the conditions for equilibrium and obtain an expression for the tension in the rope.

e) What must be the minimum tension at which the rope can break to support a 40 kg beam with length 3.0 m from which a 8.0 kg ball is suspended in the illustrated configuration?
43 Rotating disk

A 2.0 kg turntable (disk) has radius 0.10 m and can rotate horizontally about a frictionless axle through its center. A 1.2 kg blob of putty is stuck to a point on the disk three quarters of the distance from the center to the edge. A rope attached halfway from the center to the edge of the disk pulls with force 4.0 N as illustrated. The aim of this exercise is to determine the angular acceleration of the disk via the following steps.

a) Write the rotational version of Newton’s second law.
b) Determine the moment of inertia of the disk plus putty.
c) Determine the net torque acting on the disk.
d) Determine the angular acceleration of the disk.
e) Suppose that a brake pad presses on the rim of the disk, producing a frictional force with magnitude 1.5 N while the rope is pulling as before. Determine the angular acceleration of the disk in this situation.
44 Mass suspended from a rotating disk

A pulley, whose moment of inertia is $I$, can rotate about a frictionless axle through its center. Two blocks are suspended from a string, which runs, without slipping, over the pulley. The block on the left has mass $m_1$ and that on the right has mass $m_2 < m_1$. The system is held at rest and then released.

a) Write the rotational version of Newton’s second law for the pulley.

b) Determine the net torque acting on the pulley, assuming that the tension in the string on the left might be different from that on the right.

c) Determine an expression for the angular acceleration of the pulley in terms of the tensions in the strings.

d) If the moment of inertia of the pulley is non-zero and the pulley is accelerating, can the two tensions be equal?

e) Apply Newton’s second law to each of the suspended masses, in each case relating the magnitude of the acceleration to the tension in the string.

f) Combine the results from the previous parts to show that the magnitude of the acceleration of the blocks is

$$a = \frac{m_1 - m_2}{m_1 + m_2 + I/R^2} g$$

where $R$ is the radius of the pulley.
45 Vectors: Cross Products

For each of the following, determine $A_x, A_y, B_x, B_y$ and $A_z$ and $B_z$ (if applicable) and determine the cross product, $\vec{A} \times \vec{B}$. Note: If your answer for the cross product does not contain $\hat{i}, \hat{j}$ or $\hat{k}$ then it is very incorrect!

a) 
\[
\vec{A} = 2\hat{i} + 2\hat{j} \\
\vec{B} = 2\hat{i} + 2\hat{j}
\]

b) 
\[
\vec{A} = 2\hat{i} + 2\hat{j} \\
\vec{B} = 2\hat{i} - 2\hat{j}
\]

c) 
\[
\vec{A} = 2\hat{i} - 2\hat{j} \\
\vec{B} = 2\hat{i} + 2\hat{j}
\]

d) 
\[
\vec{A} = 2\hat{i} - 2\hat{j} \\
\vec{B} = 2\hat{i} - 2\hat{j}
\]

e) 
\[
\vec{A} = 2\hat{i} + 2\hat{j} + 3\hat{k} \\
\vec{B} = 2\hat{i} + 2\hat{j}
\]

f) 
\[
\vec{A} = 2\hat{i} + 2\hat{j} + 3\hat{k} \\
\vec{B} = 2\hat{i} - 2\hat{j}
\]

g) 
\[
\vec{A} = 2\hat{i} - 2\hat{j} + 3\hat{k} \\
\vec{B} = 2\hat{i} + 2\hat{j} + 3\hat{k}
\]

h) 
\[
\vec{A} = 1\hat{i} - 2\hat{j} + 3\hat{k} \\
\vec{B} = 2\hat{i} + 1\hat{j} - 3\hat{k}
\]
46 Vectors: Cross Products

For each of the following, express $\vec{A}$ in component form $\vec{B}$ using unit vectors and determine both cross products, $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$. Note: If your answer for the cross product does not contain $\hat{i}, \hat{j}$ or $\hat{k}$ then it is very incorrect!

Answer:

a) $\vec{A} \times \vec{B} = 7\hat{k}$ and $\vec{B} \times \vec{A} = -7\hat{k}$.

b) $\vec{A} \times \vec{B} = 2\hat{k}$ and $\vec{B} \times \vec{A} = -2\hat{k}$.

c) $\vec{A} \times \vec{B} = 6\hat{k}$ and $\vec{B} \times \vec{A} = -6\hat{k}$.

d) $\vec{A} \times \vec{B} = -7\hat{k}$ and $\vec{B} \times \vec{A} = 7\hat{k}$. 
47 Rotating disk

A disk with mass $M$ and radius $R$ rotates in a horizontal plane with constant angular velocity, $\omega_i$. A hoop with mass $M$ and radius $R$ is gently lowered onto the disk so that the center of the hoop coincides with the center of the disk. The hoop sticks to the disk and the two rotate with angular velocity $\omega_f$. Which of the following is true?

i) $\omega_f = \omega_i$

ii) $\omega_f = \frac{1}{2} \omega_i$

iii) $\omega_f = \frac{1}{\sqrt{2}} \omega_i$

iv) $\omega_f = \frac{1}{3} \omega_i$

v) $\omega_f = \frac{1}{\sqrt{3}} \omega_i$

Explain your answer.

48 Elliptical orbit of a satellite

A satellite moves around a planet in an elliptical orbit. At all times the force exerted by the planet on the satellite points directly toward the planet. The satellite’s motion can be analyzed by thinking of it as a point particle in a rotational orbit about the point where the planet is located.

a) Is the net torque (about the planet’s location) on the satellite zero or not at all times? Explain your answer.

b) Is the angular momentum (about the planet’s location) of the satellite constant at all times? Explain your answer.

c) Using angular momentum, describe whether the satellite is moving faster or slower at location A in comparison to location B.