No Advantage to Entanglement in Bit Flip Parameter Estimation

David Collins¹, Michael Frey²

(1) Physical and Environmental Sciences, Mesa State College, Grand Junction, CO, USA, (2) Mathematics, Bucknell University, Lewisburg, PA, USA.

Abstract

We consider optimal estimation of the parameter describing a bit-flip channel. Using the quantum Fisher information as a measure of the accuracy of the parameter estimation, we show that entanglement offers no advantage for multiple uses of the channel. This contrasts with parameter estimation in depolarizing channels, where entanglement offers a modest advantage and unitary channels where entanglement offers a distinct advantage.

Parameter estimates fluctuate statistically between repeated runs, each with \(N\) quantum operations, and is attained with entangled states.

Cramer-Rao bound and Fisher Information

The accuracy of the measurement is quantified in terms of the mean square error,

\[
\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (\lambda_i - \lambda_{\text{true}})^2
\]

For any unbiased estimator, the Cramer-Rao bound gives [1]:

\[
\text{MSE} \geq \frac{1}{\text{Var}(\hat{\lambda})} = (\lambda_{\text{true}} - \lambda)^2
\]

The quantum Fisher information is given by

\[
\mathcal{F}(\hat{\lambda}) = \text{Tr} \left( \frac{\partial}{\partial \lambda} H(\hat{\lambda}) \right) \left( \frac{\partial}{\partial \lambda} H(\hat{\lambda}) \right)^{\dagger}
\]

Optimal estimation: choose input state and additional parameter-independent unitaries so as to maximize the quantum Fisher information.

Task: Knowing the type of evolution, estimate parameters as accurately as possible by subjecting quantum systems to the evolution.

The probabilistic nature of outcomes of measurements on quantum systems and the effects of measurements on quantum states imply that the quantum operation, \(\hat{\Gamma}(\hat{\lambda})\), must be invoked repeatedly (\(N\) times).

Circuit Diagram:

Initial State \(\hat{\rho}_i\) \(\rightarrow\) Evolution \(\hat{\Gamma}(\hat{\lambda})\) \(\rightarrow\) Final State \(\hat{\rho}(\hat{\lambda})\) \(\rightarrow\) Measurement

Estimate via estimator (function of measurement outcomes):

\[
\hat{\lambda}_{\text{est}} = \lambda_{\text{true}} + \Delta \hat{\lambda}_{\text{est}}
\]

Parameter Estimation for Other Non-Unitary Channels

For programmable channels (e.g., bit-flip, phase-flip, and depolarizing channels) [2]:

- the optimal quantum Fisher information with \(N\) channel uses scales as \(O(N)\), but
- for the depolarizing channel entanglement offers advantages [4, 5, 6].

Entanglement offers advantages for depolarizing channel parameter estimation but these do not scale beyond \(O(N)\).

Conclusions

The accuracy of channel parameter estimation can be assessed via the optimal quantum Fisher information, \(\mathcal{F}(\hat{\lambda})\). For \(N\) uses of:

- the bit-flip channel, \(\mathcal{F}(\hat{\lambda}) = N(1 - \lambda)\)
- a unitary channel, \(\mathcal{F}(\hat{\lambda})\) scales as \(O(\sqrt{N})\) and is attained with entangled states.
- the depolarizing channel, \(\mathcal{F}(\hat{\lambda})\) scales as \(O(\sqrt{N})\) and entangled states offer advantages.

References