Correlated quantum states and enhanced mixed state Pauli channel parameter estimation

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Abstract

The accuracy of any physical scheme used to estimate parameters that govern the evolution of quantum systems is limited by statistical fluctuations inherent in quantum measurement processes. Quantum estimation theory provides methods for quantitative comparison of various estimation protocols. We focus on estimating the parameter governing the single qubit Pauli channel, for which it is known that optimal pure state estimation uses unentangled initial states. We consider a restricted version of this problem in which the initial states of the individual quantum systems are not pure and ask whether there are quantum estimation protocols which can yield greater accuracy than independent channel use protocols, analogous to classical repetition and averaging schemes. We compare a protocol involving quantum correlated states to independent channel use protocols. We show that, for all the pure states considered, the quantum correlated state protocol can yield greater estimation accuracy than any independent state protocol. We also show that, for all correlated states, the quantum correlated state protocol does not saturate the absolute upper bound on the QFI.

Pauli Channel Parameter Estimation

In any estimation protocol using channels involving the QFI is bounded by [4, 5, 6]:

\[ H(\lambda) \leq \frac{m}{2 \lambda + 1} \]

For pure input states there is a protocol, in which the Pauli channel is applied independently to qubits and which saturates the bound. For pure input states, entanglement and quantum correlations cannot enhance parameter estimation accuracy.

Could correlated states enhance estimation accuracy when none of the qubits are initially in pure states?

Independent Channel Use Protocol

Assume that each qubit initially has the same polarization and that the channel is a phase flip channel.

Channel inversion on m qubits.

The optimal input states all have \( r = 2 \). Then for \( m \) independent channel uses, the optimal QFI is:

\[ \lambda_{\text{opt ind}}(\lambda) = \frac{4r^2 m}{1 - (2\lambda)^2 r^2} \]

The optimal independent channel use QFI depends monotonically on \( m \) and does not saturate the absolute upper bound on the QFI.

Correlated State Protocol

Channel inversion on \( m \) of \( n \) qubits is preceded by a correlating preparatory unitary.

Preparatory unitary structure.

Same product input states as for independent channel use protocol. Here \( \hat{R} \) is a single qubit Hadamard transfromation.

\[ m \lambda \frac{1}{2} + 2m \lambda \frac{1}{2} \sum_{j=1}^{n} c_j \frac{c_j}{c_j} \]

where

\[ c_j = (1 + r)(1 - r)^{n-1} \pm (1 + r)^{n-1}(1 - r)^j. \]

Quantifying Estimation Accuracy: Fisher Information

The accuracy of the measurement is quantified in terms of the mean square error,

\[ \text{MSE} = \mathbb{E}[(\hat{\lambda} - \lambda)^2] \]

For any unbiased estimator, the Cramér–Rao bound gives [1]:

\[ \text{MSE} \geq \frac{1}{\text{Var} \{ \hat{\lambda} \}} \]

Quantum Fisher information (QFI) depends on initial state but not estimator.

Classical Fisher information depends on measurement choice but not estimator.

The quantum Fisher information is given by

\[ H(\lambda) = \text{Tr} \left( \hat{J} \hat{L} \right) \]

where

\[ \hat{J} = \frac{\partial}{\partial \lambda} \hat{\rho} = \frac{1}{2} \left( \hat{L} \dot{L} + \dot{L}\hat{L} \right) \]

Optimal estimation: choose input state and additional parameter-independent unitaries so as to maximize the quantum Fisher information.

The quantum Fisher information can be increased by invoking the channel multiple times. If there are \( m \) invocations on identically prepared independent or uncorrelated quantum systems, then it is always true that \( H(\lambda) = m H(\lambda) \) where \( H(\lambda) \) is the quantum Fisher information for one invocation on one system.

Quantum estimation: Given \( m \) channel invocations it is possible, by using quantum resources, to exceed the \( m \)-fold increase attained with "classical" independent uses of the channel.

The independent channel use improvement in the QFI can be enhanced by an additional factor of \( m \) for certain scenarios by using entangled or correlated states [2, 3].

References